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MATHEMATICS 10

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UNIT

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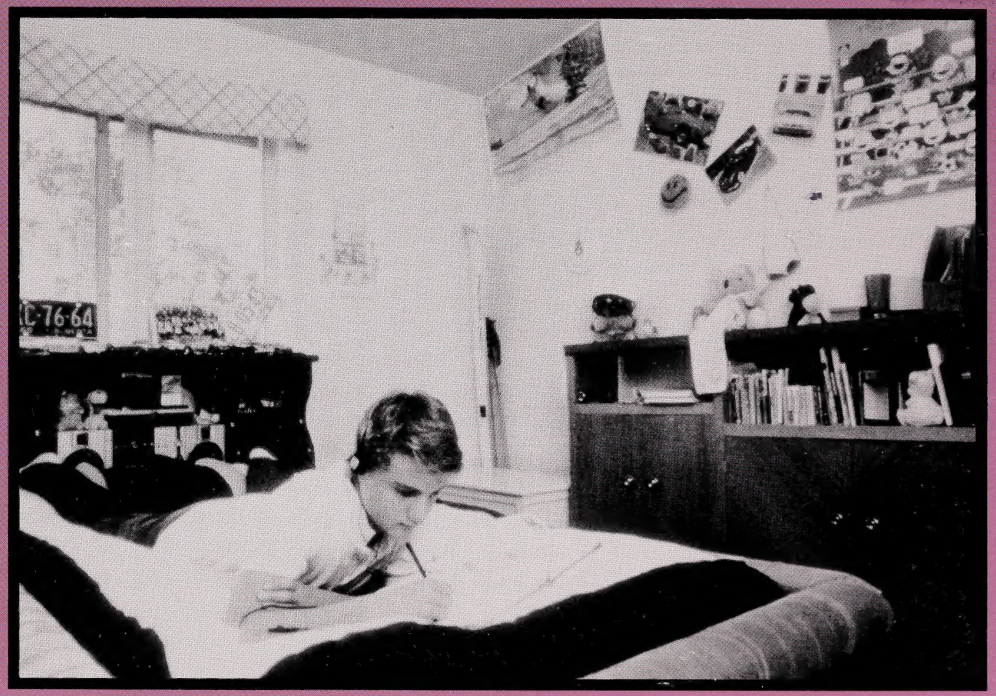

FACTORING POLYNOMIALS

Distance Learning

Alberta EDUCATION

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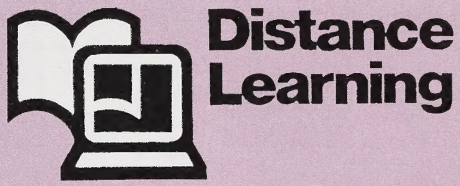
Welcome



You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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Mathematics 10 Student Module Unit 4 Factoring Polynomials Alberta Distance Learning Centre ISBN No. 0-7741-0749-9

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General Information

This information explains the basic layout of each booklet.

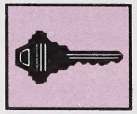
- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

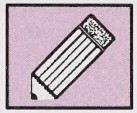
An explanation of what they mean is written beside each visual cue.

What You Already Know
- reviewing what you already know



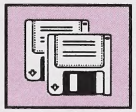
Key Idea
- flagging important ideas

Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts

Another View
- exploring different perspectives

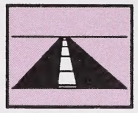


Computer Software
- learning by using computer software

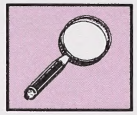
Introduction
- introducing the unit

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape



What Lies Ahead
- previewing the unit



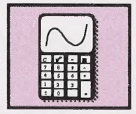
Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

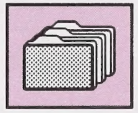
Exploring the Topic
- actively learning new concepts

Extensions
- going on with the topic

Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator



What You Have Learned
- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	**13%**
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Equations	10%
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Factoring Polynomials

This unit covers topics dealing with factoring polynomials. Each topic contains explanations, examples, and activities to assist you in understanding factoring polynomials. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 4 Factoring Polynomials

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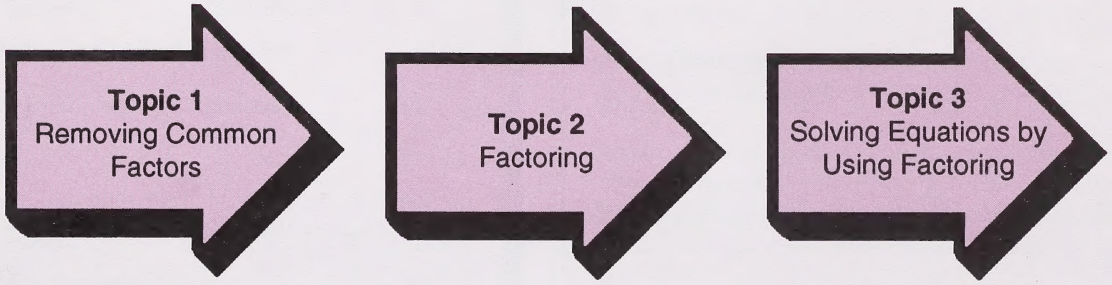
Factoring Polynomials

Within each of the following pairs, one is the opposite of the other: open and close, on and off, button and unbutton. In fact, one undoes the other. Add and subtract, and multiply and divide are examples of mathematical opposites. One operation undoes the other.

Expansion of polynomials is an operation of multiplication and simplification of polynomials. The opposite of this operation is called factoring. Factoring undoes expansion.

In this unit, you will investigate some factoring techniques and their applications when solving problems.

Unit 4
Factoring Polynomials



What You Already Know

Recall the following:

- Variables are symbols or letters that represent values (numbers). If you are given the values for the variables in an expression, you can evaluate the expression. This concept is often used to check your work in polynomials and to solve problems.

Look at this example.

Example 1

Evaluate $3y+2z$ when $y=2$ and $z=5$.

Solution:

$$\begin{aligned} 3y+2z &= 3(2)+2(5) \\ &= 6+10 \\ &= 16 \end{aligned}$$

- The power laws for exponents are required in order to simplify polynomial expressions.

$$a^m a^n = a^{m+n}$$

$$\begin{aligned} x^3 x^4 &= x^{3+4} \\ &= x^7 \end{aligned}$$

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{or} \quad a^m \div a^n = a^{m-n}$$

$$\begin{aligned} \frac{y^7}{y^4} &= y^{7-4} \\ &= y^3 \end{aligned}$$

$$\left(a^m\right)^n = a^{m \times n}$$

$$\begin{aligned} \left(x^4\right)^3 &= x^{4 \times 3} \\ &= x^{12} \end{aligned}$$

$$a^0 = 1 \text{ where } a \neq 0$$

$$x^0 = 1 \text{ where } x \neq 0$$

$$a^{-m} = \frac{1}{a^m} \text{ and } a^m = \frac{1}{a^{-m}}$$

$$\frac{x^{-5}}{y^{-3}} = \frac{y^3}{x^5}$$

- In order to simplify polynomials add or subtract like terms.

Like terms have the same variable(s) to the same degree.

Example 2

Simplify $2x^2+3x-5x^2+2x$.

Solution:

$$2x^2+3x-5x^2+2x=2x^2-5x^2+3x+2x$$
$$=-3x^2+5x$$

- Factoring is the opposite of multiplication. It is very important to be able to multiply polynomials.

  – Multiply a monomial by a monomial.

  For example, multiply $(2x^2)(2xy)$.

$$(2x^2)(2xy)=(2\cdot 2)(x^2\cdot x)(y)$$
$$=4x^3y$$

  – Multiply a polynomial by a monomial.

  For example, multiply $3x^2\left(2x^2y-x^2y^2+2xy^3\right)$.

$$3x^2\left(2x^2y-x^2y^2+2xy^3\right)=3x^2\left(2x^2y\right)-3x^2\left(x^2y^2\right)+3x^2\left(2xy^3\right)$$
$$=6x^4y-3x^4y^2+6x^3y^3$$

(Use the distributive property.)

The distributive property is used to multiply binomials.

$$(a+b)(c+d)=a(c+d)+b(c+d)$$
$$=ac+ad+bc+bd$$

– Multiply a binomial by a binomial.

For example, multiply $(2x-y)(3x^2-2xy)$ and simplify.

$$(2x-y)(3x^2-2xy) = 6x^3-4x^2y-3x^2y+2xy^2 \quad \text{(Use the distributive property.)}$$
$$= 6x^3-7x^2y+2xy^2 \quad \text{(Simplify by combining like terms.)}$$

- The greatest common factor of a set of terms is the largest term that will divide evenly into every term of the set.

Example 3

Find the greatest common factor of the following terms.

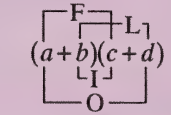
$21x^2y^3z,\ 14xy^4,\ 35x^2y^5z^2.$

Solution:

The greatest common factor is $7xy^3$. This is the largest term that divides evenly into all three terms.

Now that you have looked at some of the material you studied previously, go to the **Review** and see if you understand this material.

First terms
Outside terms
Inside terms
Last terms



Review

Try the following review questions.

1. Evaluate $3x^2 - 2y^2$ when $x = 3$ and $y = -4$.

2. Simplify each of the following completely. Only positive exponents should be in your answers.

   a. $\left(2a^2b^{-3}\right)\left(-3ab^2\right)$

   b. $\dfrac{4x^2y^5}{-2x^4y^2}$

   c. $\left(-2x^2y^{-3}\right)^2$

   d. $\left(-3x^{-2}y\right)^0$

   e. $2x^{-3}$

   f. $\dfrac{9}{3x^{-2}}$

3. Expand each of the following and collect like terms.

   a. $2x^2\left(y^3 - 3x\right)$

   b. $(x-3)(x+2)$

   c. $(x-2)^2$

   d. $(3x-y)(2y-x)$

4. Find the greatest common factor for the following.

   a. $27xy^3 + 15x^2y^2$

   b. $5x^3yz^3 - 7x^4y^2z^2 + 9x^2y^3z^3$

Now go to the **Review** solutions in **Appendix A**.

If you had trouble with this material, review **Mathematics 10, Unit 2**.

Remember: The greatest common factor is the largest term that will divide evenly into every term.

Topic 1 Removing Common Factors

Introduction

Why would mathematicians need a skill like factoring? Factoring is very useful when solving certain equations and problems that involve polynomials. It makes the task easier.

Since factoring is a tool that can be used to help solve problems, you must learn to use this tool.

If you have access to a videocassette recorder, you may view the video titled *Freddie Factor*. This program will introduce you to factoring using polynomial tiles.

What Lies Ahead

Throughout the topic you will learn to

1. factor a polynomial that has a common monomial factor
2. factor a polynomial that has a common binomial factor

Now that you know what to expect, turn the page to begin your study of removing common factors.

Exploring Topic 1

Activity 1

Factor a polynomial that has a common monomial factor.

The greatest common factor of a set of terms is the largest term that will divide evenly into every term of the set.

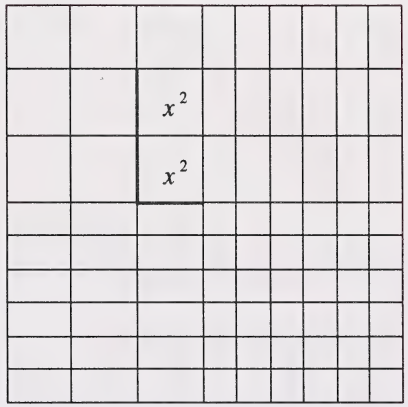
For simple terms with one variable and degree of 2 or less, you can use a binomial grid to find the greatest common factor.

Example 1

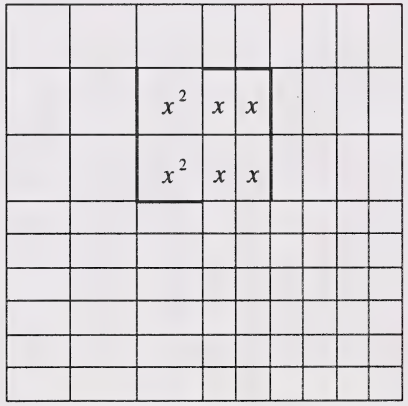
Factor $2x^2 + 4x$ using a binomial grid.

Solution:

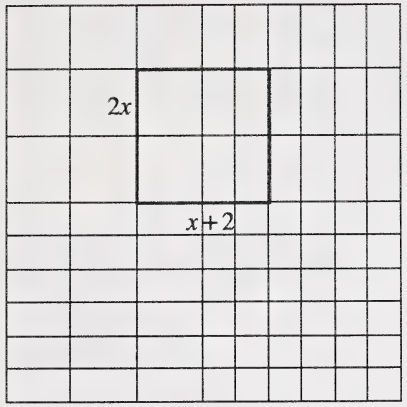
Draw a rectangle on the grid. Start with one corner containing $2x^2$.



Draw the other corner containing $4x$.



Complete the rectangle.



The dimensions of the rectangle drawn are $2x$ and $x+2$.

Therefore, the factored form of $2x^2+4x$ is $2x(x+2)$.

You may also use algebra tiles to help you factor simple polynomials with one variable of degree 2 or less.

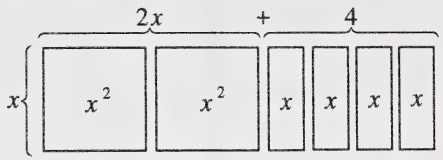
Example 2

Factor $2x^2+4x$ using algebra tiles.

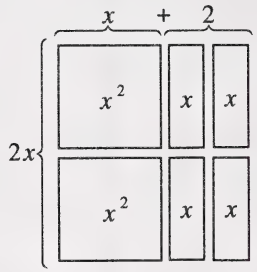
Solution:

You will need two x^2-tiles and four x-tiles.

Two rectangular arrangements of the tiles are possible.



The dimensions are x by $(2x+4)$. Therefore, the factored form of $2x^2+4x$ is $x(2x+4)$



The dimensions are $2x$ by $(x+2)$. Therefore, the factored form is $2x(x+2)$

Both answers are correct, but the rectangle that more closely resembles a square gives the complete factorization.

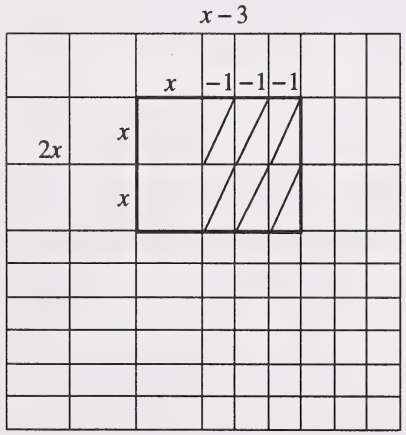
Therefore, the factored form of $2x^2+4x$ is $2x(x+2)$.

Example 3

Factor $2x^2 - 6x$ using a binomial grid.

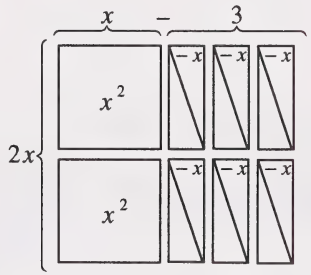
Solution:

Draw the rectangle $2x^2 - 6x$.



The dimensions are $2x$ and $x-3$; therefore, $(2x)(x-3)$ are the factors.

The solution using algebra tiles is as follows.



$2x^2 - 6x = 2x(x-3)$

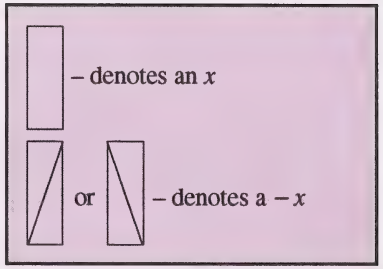
Try your luck on factoring the following binomials.

1. Factor any two of the following using a binomial grid or algebra tiles. Binomial grids and algebra tiles are provided in **Appendix B**. Cut out the algebra tiles as you need them.

   a. $6x-8$ b. $3x^2+12x$

   c. x^2-4x d. $2x-4$

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.



Binomial grids and algebra tiles are useful as concrete examples to help you better understand factoring. Since many more complex polynomials cannot be factored this way, a more thorough understanding of factoring will be required. Carefully work through the following examples.

Example 4

Find the greatest common factor of the following terms.

$21x^3,\ 7x^2,\ 35x^4$

Solution:

The greatest common factor is $7x^2$. This is the largest term that will divide evenly into all three terms.

$$\frac{21x^3}{7x^2} = 3x$$

$$\frac{7x^2}{7x^2} = 1$$

$$\frac{35x^4}{7x^2} = 5x^2$$

The first step in factoring polynomials is to identify and factor out the greatest common factor.

Example 5

Factor $2x^2 + 4y$.

Solution:

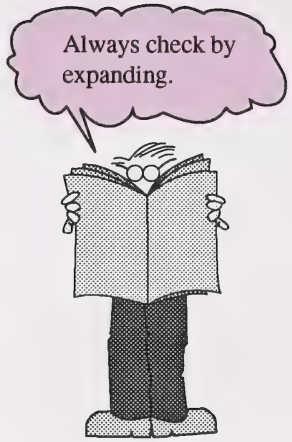
The greatest common factor is 2.

Therefore, $2x^2 + 4y = 2\left(x^2 + 2y\right)$.

Check your answer.

$$2\left(x^2 + 2y\right) = 2\left(x^2\right) + 2(2y)$$
$$= 2x^2 + 4y$$

When factoring, you are required to factor completely. In the case of the comon factor, you must factor the greatest common factor (G.C.F.). Often you may factor a polynomial, but factoring will not be complete because there is a larger common factor to be found.



Example 6

Completely factor $8x^2 + 16xy$.

Solution:

Factor 2 out of both terms.

$$2(4x^2 + 8xy)$$

You can still factor $4x^2 + 8xy$; therefore, the original question is not answered completely.

$$2(4x^2 + 8xy) = (2)(2)(2x^2 + 4xy)$$

You can still factor $(2x^2 + 4xy)$.

$$(2)(2)(2x^2 + 4xy) = (2)(2)(2)(x^2 + 2xy)$$

Can you factor $x^2 + 2xy$?

Yes, since x is a common factor.

$$(2)(2)(2)(x^2 + 2xy) = (2)(2)(2)(x)(x + 2y)$$

The complete factorization of $8x^2 + 16xy$ is $8x(x + 2y)$.

Check your answer.

$$8x(x + 2y) = 8x^2 + 16xy$$

By now you should have a good idea of what is meant by the phrase **to factor a polynomial**. To factor a polynomial is to express it as a product of factors. In the previous example, $8x$ and $(x + 2y)$ are factors of $8x^2 + 16xy$. When $8x$ and $(x + 2y)$ are multiplied, they will result in the polynomial $8x^2 + 16xy$. Also, $8x$ is a factor of $8x^2 + 16xy$ because it is the largest factor that divides evanly into $8x^2 + 16xy$.

$8x$ is the greatest common factor.

For additional work on common factors, you may use the computer program titled *Algebra Drill and Practice II*[1], disk 2, section 1. Select questions of types 1 and 2 only at this point. Do as many questions as you feel are necessary.

$x + 2y$ cannot be factored.

[1] Algebra Drill and Practice II is a title of *Conduit.*

Now try these.

2. Factor each completely and check by expanding. Do at least questions 2a to 2e.

   a. $3x+9$

   b. $8y-6$

   c. x^2+xy

   d. ax^2+ay

   e. $21x^2+14x+7$

   f. $4x^3y+8x^2y^2-4xy^3$

   g. $6x^4y^4+9x^3y^3-3x^2y^2$

   h. $25x^3yz^2-10x^2y^2z+15xy^3z^3$

3. The area of a rectangle is $16x^2-32x$. The length of one side is $8x$. By factoring, determine the length of the other side.

For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

Activity 2

Factor a polynomial that has a common binomial factor.

A polynomial may have a common factor that is a binomial. The following examples will demonstrate a binomial common factor that is immediately visible.

A binomial has two terms.

Example 7

Factor $m(s+t)+n(s+t)$.

Solution:

Each term, $m(s+t)$ and $n(s+t)$, has a common binomial factor $(s+t)$.

In Example 7, $(s+t)$ is a common factor.

$$m(s+t)+n(s+t)=(s+t)(m+n)$$

You can verify the solution by expansion.

$$(s+t)(m+n)=m(s+t)+n(s+t)$$

Notice that the expressions could have been expanded further to give the same polynomial.

$m(s+t)+n(s+t)=ms+mt+ns+nt$ and

$$(s+t)(m+n)=ms+ns+mt+nt$$
$$=ms+mt+ns+nt$$

Example 8

Factor $x^2(y+1)+5x(y+1)+7(y+1)$.

Solution:

$(y+1)$ is a common factor; therefore, $x^2(y+1)+5x(y+1)+7(y+1)$ is equal to $(y+1)(x^2+5x+7)$.

Check your answer.

$$(y+1)(x^2+5x+7)$$
$$=x^2(y+1)+5x(y+1)+7(y+1)$$

Some polynomials can be factored by first factoring a common term from groups of two terms.

Example 9

Factor $ms+mt+ns+nt$.

Solution:

Group the terms in pairs so that each member of the pair has a common factor. Then factor the common terms m and n from each of the two groups respectively.

$$(ms+mt)+(ns+nt)=m(s+t)+n(s+t)$$
$$=(m+n)(s+t)$$

Check your answer.

$$(m+n)(s+t)=ms+mt+ns+nt$$

You may have to regroup the terms before you can see the common factors.

Use FOIL when expanding binomials.

Example 10

Factor $mc+3nd+md+3nc$.

Solution:

Group the terms that have common factors.

$$mc+md+3nc+3nd$$

Now factor the common factors.

$$\begin{aligned}(mc+md)+(3nc+3nd)\\ &= m(c+d)+3n(c+d)\\ &= (m+3n)(c+d)\end{aligned}$$

Check your answer.

$$\begin{aligned}(m+3n)(c+d) &= mc+md+3nc+3nd\\ &= mc+3nd+md+3nc\end{aligned}$$

Example 11

Factor $b^3-21+7b-3b^2$.

Solution:

Regroup.

$$b^3-3b^2+7b-21$$

Factor the groups.

$$b^2(b-3)+7(b-3)$$

Factor completely.

$$(b-3)\left(b^2+7\right)$$

Check your answer.

$$\begin{aligned}(b-3)\left(b^2+7\right) &= b^3+7b-3b^2-21\\ &= b^3-21+7b-3b^2\end{aligned}$$

In Example 10, you could have grouped $(mc+3nc)$ and $(3nd+md)$

$$\begin{aligned}(mc+3nc)+(3nd+md)\\ &= c(m+3n)+d(3n+m)\\ &= (m+3n)(c+d)\end{aligned}$$

This time the common factor is $(m+3n)$.

In Example 11, the equation was factored twice. First factor a common factor from each group; then factor the common binomial from each group.

Do at least six questions.

Factor completely by grouping.

1. $ab^2 - b^2 - 2a + 2$
2. $dx + dy + 2bx + 2by$
3. $am^2 + amn - km - kn$
4. $yb - b - 2y + 2$
5. $a^3 - y - a^2 + ay$
6. $6 - 3a - 2b + ab$
7. $3m^3 + 2m + 12m^2 + 8$
8. $2m^3 + 2m - 3m^2 - 3$
9. $ae + bd - ad - be$
10. $ab + 3a - b - 3$

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

The basic steps in factoring polynomials are as follows.

- Factor out the greatest common monomial factor from each term.
- Factor out the common monomial factor from each group of terms. (You may first have to regroup the terms)
- Factor out the common binomial factor.

Example 12

Factor $3ax+3ay+3bx+3by$.

Solution:

Factor out the greatest common monomial factor from each term.

$$3ax+3ay+3bx+3by=3(ax+ay+bx+by)$$

Factor out the common monomial factors from each groups of term.

$$\begin{aligned}3(ax+ay+bx+by)&=3\left[(ax+ay)+(bx+by)\right]\\&=3\left[a(x+y)+b(x+y)\right]\end{aligned}$$

Factor out any common binomial factor.

$$\begin{aligned}3\left[a(x+y)+b(x+y)\right]&=3\left[(x+y)(a+b)\right]\\&=3(x+y)(a+b)\end{aligned}$$

Try some of the following questions.

Factor completely.

1. $6ce+6de+6cf+6df$
2. $4acx+4adx+4bcx+4bdx$
3. $6a^2b+9axy+6abx+9x^2y$

For solutions to **Extra Help**, turn to **Appendix A**, **Topic 1**.

Extensions

The same rules and principles of factoring are applied to more complex polynomials.

Example 13

Factor completely $ace + bcf + ade + bce + acf + bde + adf + bdf$.

Solution:

Both a and b are common factors in four terms each; thus, rearrange the terms.

$$\begin{aligned} ace + bcf + ade + bce + acf + bde + adf + bdf &= ace + ade + acf + adf + bcf + bce + bde + bdf \\ &= a(ce + de + cf + df) + b(cf + ce + de + df) \\ &= a(ce + cf + de + df) + b(ce + cf + de + df) \\ &= a[c(e + f) + d(e + f)] + b[c(e + f) + d(e + f)] \\ &= a[(e + f)(c + d)] + b[(e + f)(c + d)] \\ &= (e + f)(c + d)(a + b) \end{aligned}$$

After a and b are factored out, the terms inside the parentheses may be rearranged so the terms in the common factors are in the same order.

Try some of these problems. Factor completely.

1. $2a^2 - 3a + 3b - 2ab + 2ac - 3c$
2. $6xy - y - 6xz + z - 6x + 1$
3. $2acx - 2ady + adx - 4acy + 2bcx + bdx - 2bdy - 4bcy$

For solutions to **Extensions**, turn to the **Appendix**, **Topic 1**.

Topic 2 Factoring

Introduction

In the previous topic you learned to factor the greatest common factor out of a polynomial.

In this topic you will factor polynomials that are more complicated. Some of the polynomials can be very challenging.

Can you factor $2x^2 - x - 15$? Example 7 in this topic will show you how.

What Lies Ahead

Throughout this topic you will learn to

1. factor trinomials of the form $ax^2 + bx + c$, where $a, b, c, \in I$
2. recognize the expansion of the square of a binomial as a special case
3. factor a perfect square trinomial
4. recognize the product of conjugate binomials as a special case
5. factor a difference of squares binomial
6. factor polynomials that require a combination of methods in order to be completely factored

Now that you know what to expect, turn the page to begin your study of factoring.

Exploring Topic 2

Activity 1

Factor trinomials of the form ax^2+bx+c, where $a, b, c, \in I$.

You will achieve the objective of this activity using small steps to make it easier to learn. The first type of trinomial you will learn to factor is of the form ax^2+bx+c, where $a=1, b, c, \in I$.

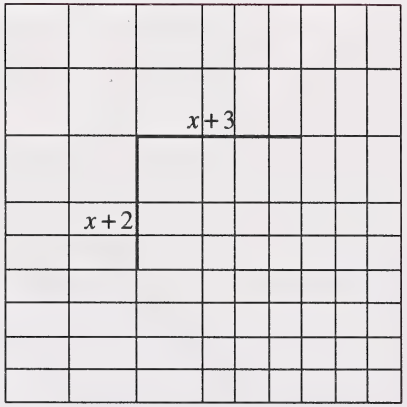
You already know how to multiply binomials using a binomial grid.

Example 1

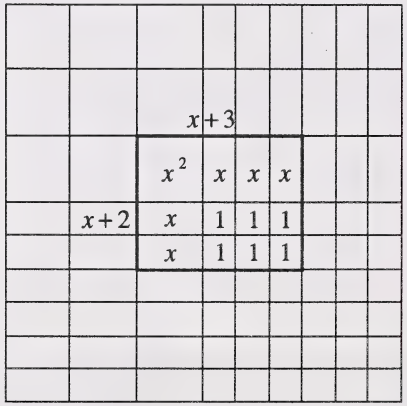
Expand $(x+2)(x+3)$ using a binomial grid.

Solution:

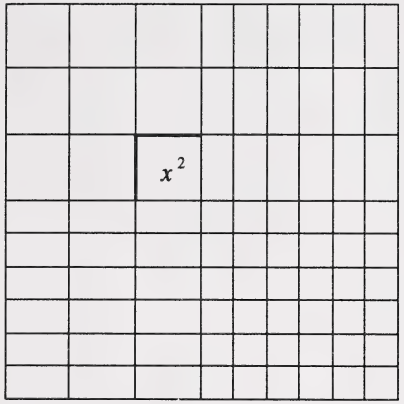
Draw $(x+2)$ and $(x+3)$ as the sides of the rectangle so that they meet at one corner.



The rectangle formed by the sides $(x+2)$ and $(x+3)$ represents the product of the binomials $(x+2)$ and $(x+3)$. According to the binomial grid, $(x+2)(x+3)=x^2+5x+6$.

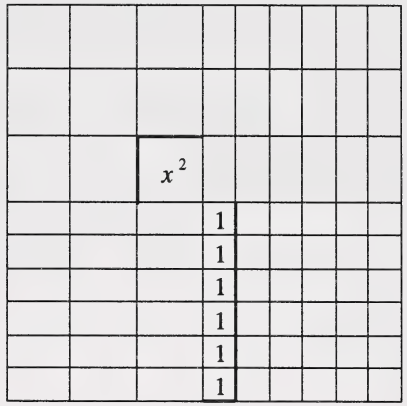


Factoring is the opposite of expansion. When you are asked to factor x^2+5x+6, you are given a trinomial and asked to find the binomial factors. The factors can be found using a binomial grid. First draw the rectangle that represents x^2+5x+6. Start by drawing the corner which will include the one x^2 term.

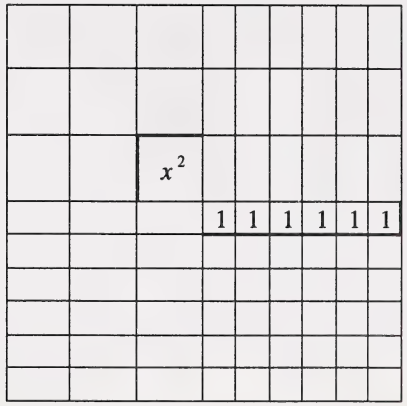


Now draw the other corner of the rectangle. It will include six units. There are four possible ways to do this.

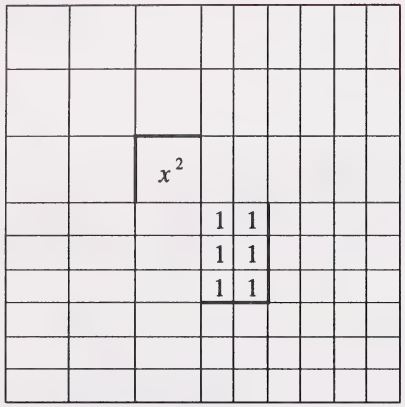
- Grid 1



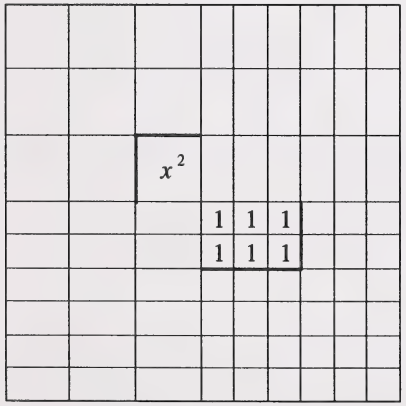
- Grid 2



- Grid 3

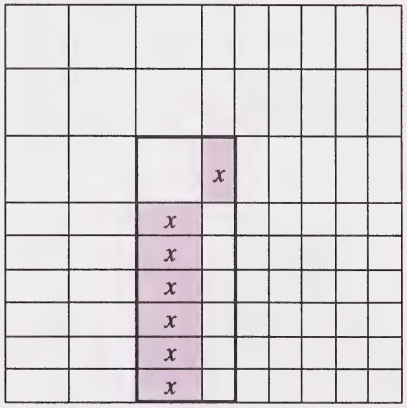


- Grid 4

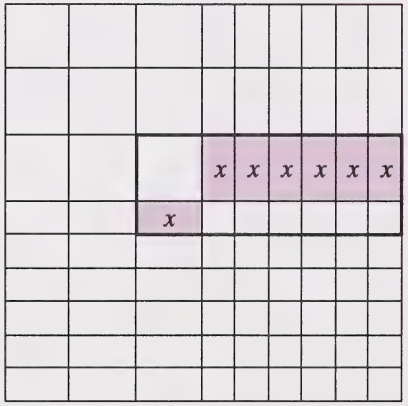


Now complete the sides of the rectangles.

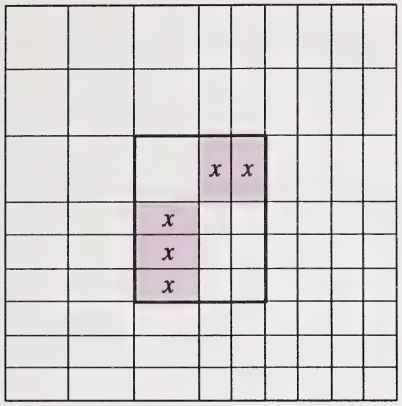
- Grid 1



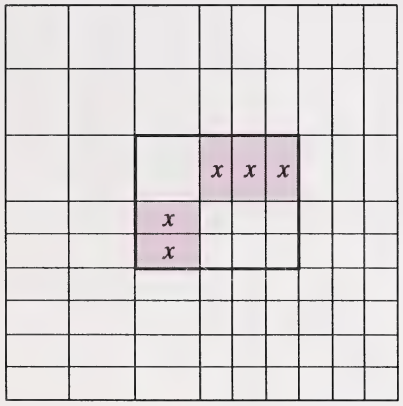
- Grid 2



- Grid 3



- Grid 4



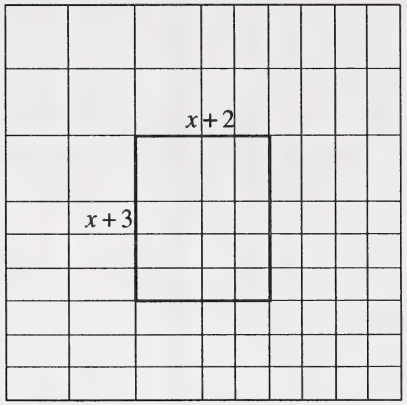
Now count the number of x-terms in each case. The x-terms have been labelled for you in the previous diagrams.

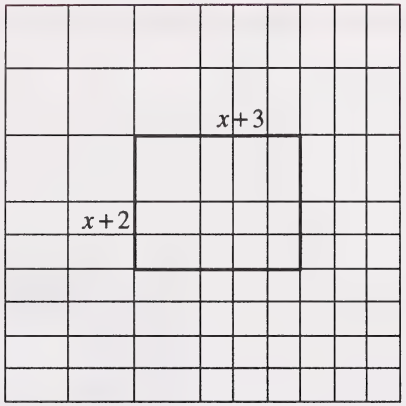
Grid 1 and Grid 2 have seven x-areas each.

Grid 3 and Grid 4 have five x-areas each.

Since the middle term of the question is $5x$, the only possible solutions are Grid 3 and Grid 4.

Write in the lengths of the sides of the rectangles.



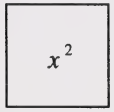


Therefore, the factors of x^2+5x+6 are $(x+3)(x+2)$ or $(x+2)(x+3)$.

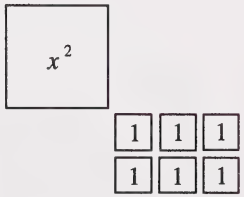
Since multiplication is commutative, the two answers are the same. Both are acceptable.

Future examples of factoring using a binomial grid will not show all possible choices, only one of the correct ways.

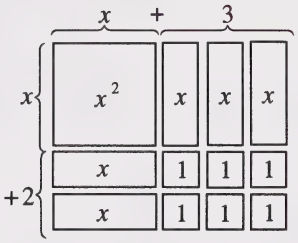
You may also use algebra tiles to help factor simple polynomials such as x^2+5x+6. Take out one x^2-tile, five x-tiles, and six 1-tiles. Make one corner of the rectangle using the x^2-tile.



Arrange the six 1-tiles as the opposite corner.



Fill in the rectangle with the x-tiles.



Therefore, the factors are $(x+2)(x+3)$.

Example 2

Factor $x^2+8x+15$ using a binomial grid.

Solution:

Draw a rectangle representing $x^2+8x+15$ on the grid. Label the length of the sides.

				x	$+5$			
		x^2	x	x	x	x	x	
	$x+3$	x	1	1	1	1	1	
		x	1	1	1	1	1	
		x	1	1	1	1	1	

Therefore, the factors are $(x+3)(x+5)$.

Check your answer.

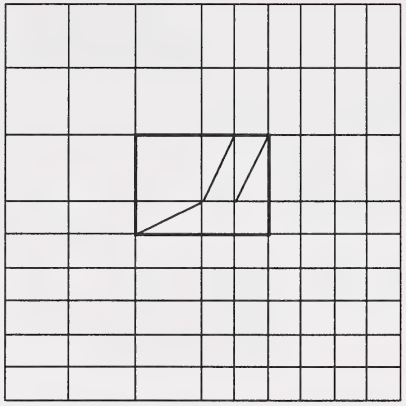
$$(x+3)(x+5) = x^2+5x+3x+15$$
$$= x^2+8x+15$$

Example 3

Factor x^2-3x+2 using a binomial grid.

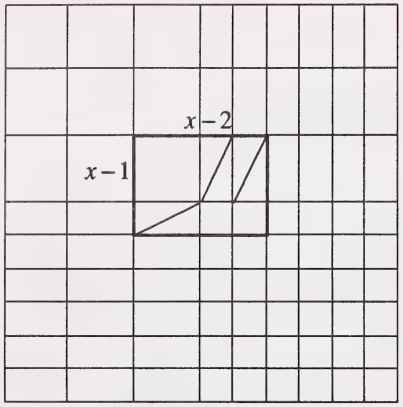
Solution:

Draw the rectangle.

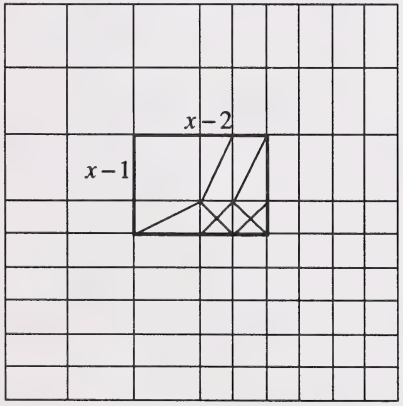


Since there are negative x areas and only positive units, the problem now is to determine the lengths of the sides. If you multiply a positive quantity by a negative quantity, you get a negative product. If you multiply a negative quantity by a negative quantity, you get a positive product. Therefore, the sides of the rectangle must have negative units.

Build this rectangle by starting with the top left corner at $1x^2$ and the bottom right corner at 15 units.



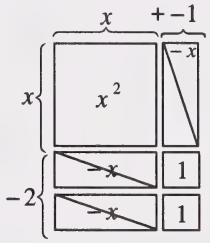
If you multiply these sides, the product would be crossed out twice giving the original positive unit areas.



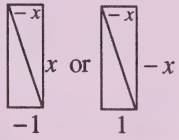
Therefore, the factors $x^2 - 3x + 2$ are $(x-1)$ and $(x-2)$.

Now try to factor $x^2 - 3x + 2$ using algebra tiles. You will need one x^2-tile, three x-tiles, and two 1-tiles. Begin with the corners made of x^2-tiles and 1-tiles.

The factored form of $x^2 - 3x + 2$ is $(x-2)(x-1)$.



Note: means that the area of this rectangle is $-x$. Its dimensions could be



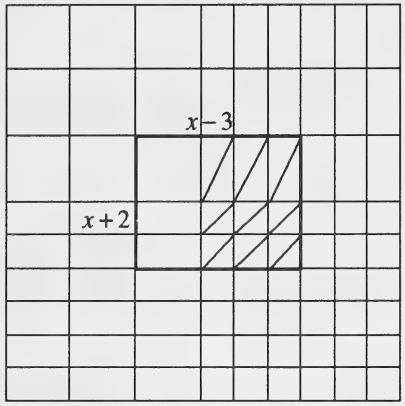
Note: also means that the area of the rectangle is $-x$. The slope of the line makes no difference.

Example 4

Factor $x^2 - x - 6$

Solution:

Draw the rectangle containing $x^2 - x - 6$.



Therefore, the factors of $x^2 - x - 6$ are $(x+2)$ and $(x-3)$.

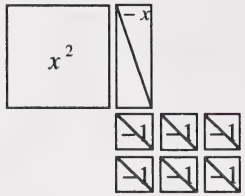
Now try it with algebra tiles.

Example 5

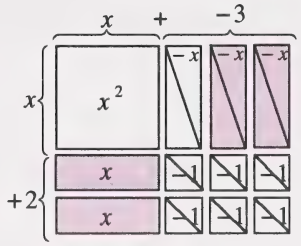
Factor $x^2 - x - 6$.

Solution:

Begin the rectangle with one x^2 in one corner and six 1-tiles in the other corner; then, fit in the x-tile.



To complete the rectangle, four x-tiles will be needed: two $-x$-tiles and two x-tiles.



$\therefore x^2 - x - 6 = (x+2)(x-3)$

Now try these. Do at least three.

1. Factor using a binomial grid or algebra tiles. Binomial grids and algebra tiles are provided in **Appendix B**.

   a. x^2+x-6 b. x^2-4x+3

   c. x^2-x-12 d. $x^2+3x-10$

   e. $x^2+7x+12$ f. x^2-5x-6

For solutions to **Activity 1**, turn to **Appendix A, Topic 2**.

Have you noticed a pattern between the factors and the product? The following examples will show you the pattern that exists and how to use it in factoring trinomials.

Expand $(x+2)(x+3)$ using the FOIL method.

$$(x+2)(x+3)=x^2+3x+2x+6$$
$$=x^2+5x+6$$

Now look at the bold face numbers.

$$(x+\mathbf{2})(x+\mathbf{3})=x^2+\mathbf{5}x+\mathbf{6}$$
$$\mathbf{2\times 3=6}$$

Multiply the last term of each binomial factor to get the last term in the trinomial.

Now look at the boldface numbers.

$$(x+\mathbf{2})(x+\mathbf{3})=x^2+\mathbf{5}x+6$$
$$\mathbf{2+3=5}$$

Add the last term of each binomial factor to get the coefficient of the middle term in the trinomial.

Look at some more examples to see if the pattern remains constant.

$(x+2)(x-3)=x^2-x-6$

$(+2)(-3)=-6$

$(+2)+(-3)=-1$

$(x+3)(x-2)=x^2+x-6$

$(+3)(-2)=-6$

$(+3)+(-2)=+1$

The pattern does remain.

Reverse the procedure and use the pattern to factor the trinomials $x^2 - x - 6$.

$$x^2 - x - 6 = (x____)(x____)$$

What two numbers give -6 when multiplied and -1 when added? Try a few pairs of intergers and select the pair that satisfies the required conditions.

$(-6)(+1) = -6$ $\qquad$ $(-1)(+6) = -6$

$(-6)+(+1) = -5$ $\qquad$ $(-1)+(+6) = +5$

(no good) $\qquad$ (no good)

$(+2)(-3) = -6$ $\qquad$ $(+2)(-3) = -6$

$(+3)+(-2) = +1$ $\qquad$ $(+2)+(-3) = -1$

(no good) $\qquad$ (This is the pair.)

Therefore, the pattern yields $x^2 - x - 6 = (x+2)(x-3)$. Another example of the process is as follows.

$$x^2 + 5x + 6 = (x\quad)(x\quad)$$

$(6)(1) = 6$ $\qquad$ $(3)(2) = 6$

$(6)+(1) = 7$ $\qquad$ $(3)+(2) = 5$

(no good) $\qquad$ (This is the pair.)

Therefore, the factors are $(x+3)(x+2)$.

Do at least seven.

2. Complete the factorization by filling in the blanks with the correct values.

   a. $x^2 - 4x + 3 = (x____)(x____)$

   b. $y^2 - 6y - 7 = (y____)(y____)$

   c. $x^2 - 4x - 32 = (x____)(x____)$

   d. $y^2 + 9y + 14 = (y____)(y____)$

   e. $x^2 - 11x + 30 = (x____)(x____)$

   f. $y^2 - 4y - 12 = (y____)(y____)$

   g. $x^2 + 2x - 15 = (x____)(x____)$

   h. $y^2 - 3y - 28 = (y____)(y____)$

   i. $x^2 - 9x + 20 = (x____)(x____)$

   j. $x^2 + 2x - 35 = (x____)(x____)$

k. $y^2 - 9y - 22 = (y____)(y____)$

l. $x^2 - 9x + 18 = (x____)(x____)$

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

Notice that the first term of each binomial factor is simply the square root of the first term of the trinomial. Actually, these terms have to multiply to give the first term in the trinomial.

Try factoring when you are not given the first terms of the factors. Do at least five.

3. Factor the followng trinomials.

a. $x^2 - 6x + 8$

b. $y^2 - 2y - 3$

c. $x^2 - 10x + 21$

d. $y^2 - 3y - 54$

e. $a^2 + 8a + 7$

f. $x^2 - 13x + 40$

g. $y^2 + 18y + 72$

h. $x^2 + 11x - 42$

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

Often a trinomial may appear to be very difficult to factor because a common factor makes the terms more complex. If possible, factor out the greatest common factor first.

Example 6

Factor $7x^2y + 35xy + 42y$.

Solution:

The greatest common factor is 7y.

$$7x^2y + 35xy + 42y = 7y\left(x^2 + 5x + 6\right)$$

Now factor the trinomial $x^2 + 5x + 6$.

The factors are $(x+2)(x+3)$.

Therefore, the complete factorization of $7x^2y + 35xy + 42y$ is $7y(x+2)(x+3)$.

Try the following. Do at least five.

4. Factor completely.

a. $-2x^2+2x+4$

b. $-x^2-4x+21$

c. $2y^2+12y+10$

d. $15x^2-45x+30$

e. $6xy^2-12xy-18x$

f. $-3x^2-24x-21$

g. $2ax^2+22ax+20a$

h. $3x^3-12x^2+12x$

i. $4x^2a^2+8x^2a-12x^2$

j. $bx^2+2bx-35b$

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

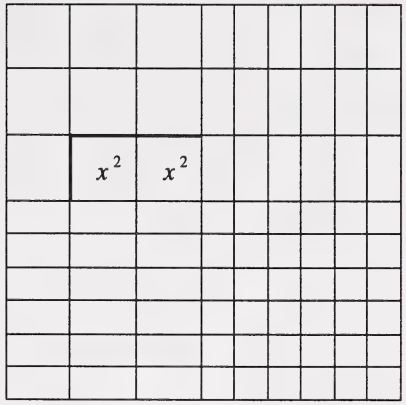
Now you will make the last step where the value of a can be the set of integers.

Look at the expansion of $(2x+1)(x+3)$.

$$(2x+1)(x+3)=2x^2+6x+x+3$$
$$=2x^2+7x+3$$

Construct this polynomial on a binomial grid.

First draw the left corner including $2x^2$.



Now draw the lower right corner including 3 units and $7x$.

ax^2+bx+c, where $a, b, c \in I$.

When expanding, use the FOIL method.

			x					
	x	x	1					
	x	x	1					
	x	x	1					

Complete the rectangle and write the lengths of the sides.

		$2x+1$						
$x+3$								

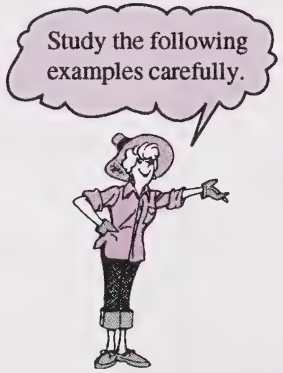
Therefore, the factors are $(x+3)$ and $(2x+1)$.

Check your answer.

$$(x+3)(2x+1) = 2x^2 + x + 6x + 3$$
$$= 2x^2 + 7x + 3$$

Using algebra tiles, take out two x^2-tiles, seven x-tiles, and three 1-tiles. Arrange the tiles into a rectangle.

	$2x$		$+1$
x	x^2	x^2	x
$+3$	x	x	1
	x	x	1
	x	x	1

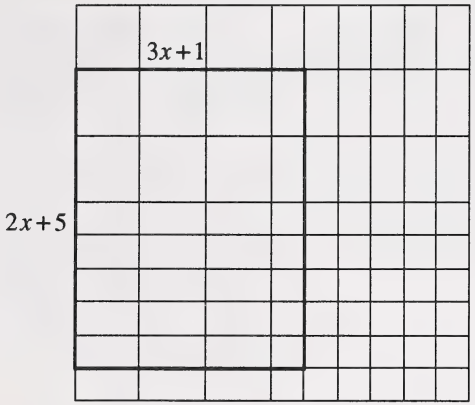


Example 7

Factor $6x^2+17x+5$ using a binomial grid.

Solution:

Draw the rectangle representing $6x^2+17x+5$, and determine the lengths of each side.



Therefore, the factors of $6x^2+17x+5$ must be $(2x+5)$ and $(3x+1)$.

Check your answer.

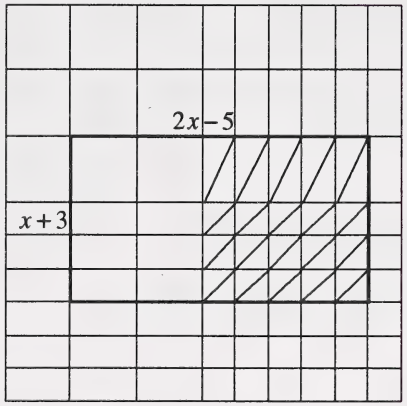
$$(2x+5)(3x+1)=6x^2+2x+15x+5$$
$$=6x^2+17x+5$$

Example 8

Factor $2x^2+x-15$ using a binomial grid.

Solution:

Draw the rectangle representing $2x^2+x-15$. The rectangle is chosen so that you can have six x-terms on one side and five on the other.

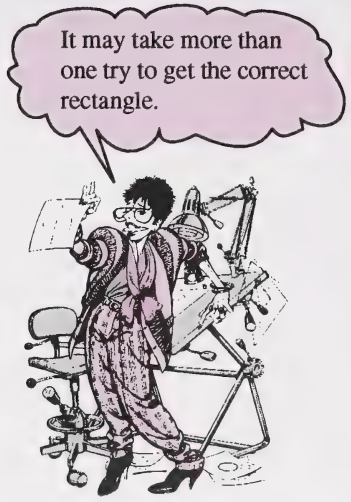


Therefore, the factors are $(2x-5)$ and $(x+3)$.

Check your answer.

$$(2x-5)(x+3)=2x^2+6x-5x-15$$
$$=2x^2+x-15$$

Examples 7 and 8 can also be solved using algrebra tiles.

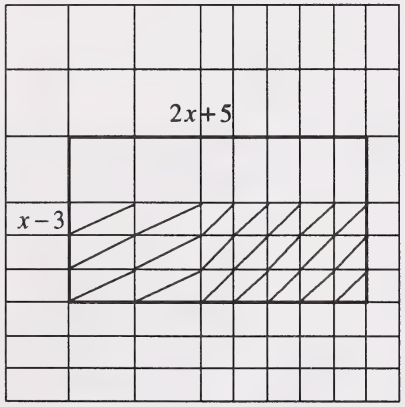


Example 9

Factor $2x^2 - x - 15$ using a binomial grid.

Solution:

Draw the rectangle representing $2x^2 - x - 15$.



Therefore, the factors are $(x-3)$ and $(2x+5)$.

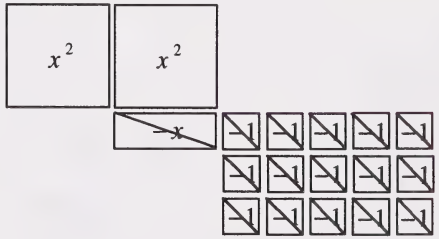
Check your answer.

$$(x-3)(2x+5) = 2x^2 + 5x - 6x - 15$$
$$= 2x^2 - x - 15$$

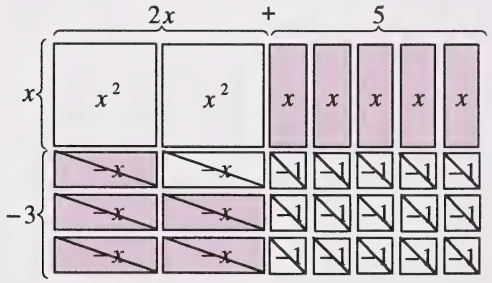
Now try to factor $2x^2 - x - 15$ using algebra tiles.

You will need two x^2-tiles, one x-tile, and fifteen 1-tiles.

Begin with the corners made up of x^2-tiles and 1-tiles. Fit in the x-tile.



To complete the rectangle, ten x-tiles are needed. To maintain the original area, add the same number of x-tiles and $-x$-tiles.



Therefore, $2x^2 - x - 15 = (x-3)(2x+5)$.

Compare Example 9 to Example 8. Only the number of negative x-terms have changed.

Recall: The coloured tiles denote tiles that are added to complete the rectangle.

Do at least five.

5. Factor using a binomial grid or algebra tiles. Binomial grids and algebra tiles are provided in **Appendix B**.

   a. $3x^2+5x+2$ b. $4x^2-4x-3$

   c. $6x^2+x-2$ d. $2x^2+5x+3$

   e. $3x^2-4x-15$ f. $2x^2-11x+12$

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

A binomial grid can be used to factor trinomials provided the numerical coefficients are not very large. To factor trinomials that contain large numerical coefficients, an algebraic method called the **cross product sum** (CPS) is used. This method can be used to factor any trinomial.

Study the following example.

$$(3x+1)(2x+5)=6x^2+15x+2x+5$$
$$=6x^2+17x+5$$

The first term in the trinomial is $6x^2$, which is the product of $3x$ and $2x$ (the first terms in the binomials).

$$(\mathbf{3x}+1)(\mathbf{2x}+5)=\mathbf{6x^2}+17x+5$$

The last term in the trinomial is 5 which is the product of 1 and 5 (the last terms in the binomials).

$$(3x+\mathbf{1})(2x+\mathbf{5})=6x^2+17x+\mathbf{5}$$

The middle term in the trinomial is $17x$, the sum of the outer and inner products ($15x$ and $2x$) formed by the binomial expansion.

$$\overset{\text{O}}{\overset{\text{I}}{(3x+1)(2x+5)}}=6x^2+\underbrace{15x+2x}_{\mathbf{17x}}+5$$

Now suppose you want to reverse the procedure and factor the trinomial $6x^2+17x+5$. Note that the first term is $6x^2$ and is a result of the product of the first terms of the binomial factors. There are two possible ways to factor $6x^2$.

$$(6x)(x)=6x^2 \text{ or } (3x)(2x)=6x^2$$

If you leave out the unknown last terms of the binomial factors, you can write the following.

$$(6x____)(x____) \text{ or } (3x____)(2x____)$$

The third term of the trinomial is 5 and is obtained by multiplying the last terms of the binomial factors. There are only two possible factors of 5.

$$(5)(1)=5 \text{ or } (-5)(-1)=5$$

The possible binomial factors are written as follows. All of the combinations have been written.

$(6x+5)(x+1)$ $(3x+5)(2x+1)$
$(6x+1)(x+5)$ $(3x+1)(2x+5)$

$(6x-5)(x-1)$ $(3x-5)(2x-1)$
$(6x-1)(x-5)$ $(3x-1)(2x-5)$

The following is a list of the previous possibilities along with the cross product sum for each.

For $(6x+5)(x+1)$, the CPS is $(6x)(1)+(5)(x)=11x$.

Possible Factors	Cross Product Sum
$(6x+5)(x+1)$	$(6x)(1)+(5)(x)=11x$
$(6x+1)(x+5)$	$(6x)(5)+(1)(x)=31x$
$(6x-5)(x-1)$	$(6x)(-1)+(-5)(x)=-11x$
$(6x-1)(x-5)$	$(6x)(-5)+(-1)(x)=-31x$
$(3x+5)(2x+1)$	$(3x)(1)+(5)(2x)=13x$
$\mathbf{(3x+1)(2x+5)}$	$\mathbf{(3x)(5)+(1)(2x)=17x}$
$(3x-5)(2x-1)$	$(3x)(-1)+(-5)(2x)=-13x$
$(3x-1)(2x-5)$	$(3x)(-5)+(-1)(2x)=-17x$

The only factors that produce the correct middle term of the trinomial are $(3x+1)(2x+5)$.

Check your answer.

$$(3x+1)(2x+5)=6x^2+15x+2x+5$$
$$=6x^2+17x+5$$

CPS = Outer + Inner

$$\overset{\ulcorner\ \text{O}\ \urcorner}{(6x+5)(x+1)}$$
$$\overset{\llcorner\ \text{I}\ \lrcorner}{}$$

Since the cross product sum equals $17x$, $(3x+1)(2x+5)$ are the factors.

There were some factors that were not chosen intelligently. Factors such as $(6x-5)(x-1)$ produce a negative CPS $(-11x)$. Therefore, all of the factors with negative values could have been excluded in order to make the list of possible factors shorter.

Here is an example involving negatives.

Example 10

Factor $12x^2-41x+15$.

Solution:

The possible first terms of the binomial factors must form the product $12x^2$.

The possible factors are $(12x)(1x)$, $(2x)(6x)$, and $(3x)(4x)$.

The binomial factors will have the following possibilities.

$(12x____)(x____)$

$(2x____)(6x____)$

$(3x____)(4x____)$

The constant of the trinomial is 15; thus, the possible factors are $(15)(1)$, $(3)(5)$, $(-15)(-1)$, and $(-3)(-5)$.

Write out all possible factors and the cross product sum.

Possible Factors	Cross Product Sum
$\mathbf{(12x+15)(x+1)}$	$(12x)(1)+(15)(x)=27x$
$\mathbf{(12x+1)(x+15)}$	$(12x)(15)+(1)(x)=181x$
$\mathbf{(12x+3)(x+5)}$	$(12x)(5)+(3)(x)=63x$
$\mathbf{(12x+5)(x+3)}$	$(12x)(3)+(5)(x)=41x$
$(12x-15)(x-1)$	$(12x)(-1)+(-15)(x)=-27x$
$(12x-1)(x-15)$	$(12x)(-15)+(-1)(x)=-181x$
$(12x-3)(x-5)$	$(12x)(-5)+(-3)(x)=-63x$
$(12x-5)(x-3)$	$(12x)(-3)+(-5)(x)=-41x$
$\mathbf{(2x+15)(6x+1)}$	$(2x)(1)+(15)(6x)=92x$
$\mathbf{(2x+1)(6x+15)}$	$(2x)(15)+(1)(6x)=36x$
$\mathbf{(2x+3)(6x+5)}$	$(2x)(5)+(3)(6x)=28x$
$\mathbf{(2x+5)(6x+3)}$	$(2x)(3)+(5)(6x)=36x$
$(2x-15)(6x-1)$	$(2x)(-1)+(-15)(6x)=-92x$
$(2x-1)(6x-15)$	$(2x)(-15)+(-1)(6x)=-36x$
$(2x-3)(6x-5)$	$(2x)(-5)+(-3)(6x)=-28x$
$(2x-5)(6x-3)$	$(2x)(-3)+(-5)(6x)=-36x$
$\mathbf{(3x+15)(4x+1)}$	$(3x)(1)+(15)(4x)=63x$
$\mathbf{(3x+1)(4x+15)}$	$(3x)(15)+(1)(4x)=49x$
$\mathbf{(3x+3)(4x+5)}$	$(3x)(5)+(3)(4x)=27x$
$\mathbf{(3x+5)(4x+3)}$	$(3x)(3)+(5)(4x)=29x$
$(3x-15)(4x-1)$	$(3x)(-1)+(-15)(4x)=-63x$
$(3x-1)(4x-15)$	$(3x)(-15)+(-1)(4x)=-49x$
$(3x-3)(4x-5)$	$(3x)(-5)+(-3)(4x)=-27x$
$(3x-5)(4x-3)$	$(3x)(-3)+(-5)(4x)=-29x$

The only factors that produce the correct cross product sum are $(12x-5)$ and $(x-3)$.

Since the CPS is negative $(-41x)$, those factors with positive second terms could have been left out in order to make the list shorter.

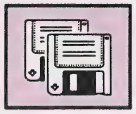
Therefore,

$12x^2 - 41x + 15 = (12x - 5)(x - 3)$.

Check our answer.

$$(12x-5)(x-3) = 12x^2 - 36x - 5x + 15$$
$$= 12x^2 - 41x + 15$$

In this example all possibilities were shown. If you were doing the factoring, you would have stopped as soon as you found the correct factors. Use your mathematical sense to try to pick those factors that are most likely to produce the cross product sum.



For additonal help with factoring trinomials, use the computer program titled *Algebra Drill and Practice II*[1], disk 2, section 1. Select questions of types 3 and 4 at this point. Do as many questions you feel necessary.

Now try the following exercise. Do at least eight.

6. Factor the trinomials.

a. $2x^2 + 13x + 15$

b. $12x^2 - 29x + 15$

c. $8x^2 + 10x - 3$

d. $10x^2 - 29x + 10$

e. $15x^2 - 29x - 14$

f. $8x^2 + 14x - 15$

g. $8x^2 + 8x - 30$

h. $6x^2 + 7x - 5$

i. $9x^2 - 36x + 27$

j. $2x^2 + x - 15$

k. $4x^2 - 17x - 15$

l. $9x^2 + 9x - 10$

For solutions to **Activity 1**, turn to **Appendix A, Topic 2**.

[1] Algebra Drill and Practice II is a title of *Conduit*.

Do not forget to look for common factors.

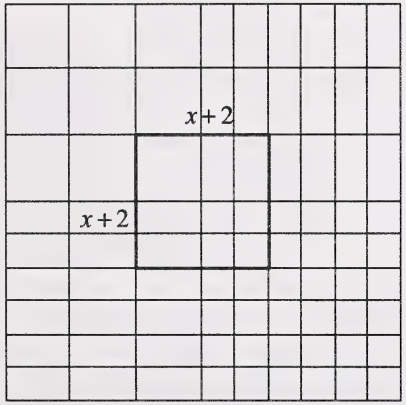
List all the possible factors until you find the ones which produce the correct cross product sum.

If you are have difficulty with factoring these trinomials, turn to the **Extra Help** section.

Activity 2

Recognize the expansion of the square of a binomial as a special case.

First examine the product of $(x+2)^2$ or $(x+2)(x+2)$ using a binomial grid.



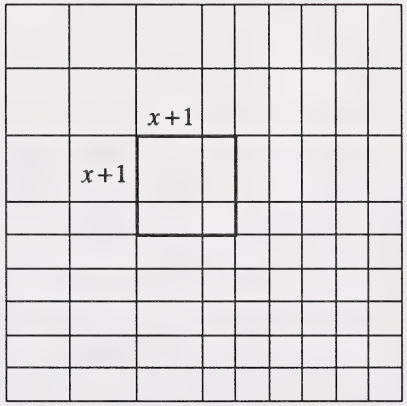
The trinomial produced can be represented by a square area on a binomial grid. This fact is used to recognize a trinomial as a perfect square trinomial.

Is x^2+2x+1 a perfect square trinomial?

$$x^2+2x+1=(x+1)(x+1)$$

Therefore, x^2+2x+1 is a perfect square trinomial.

Draw x^2+2x+1 on a binomial grid.



The square of a binomial means a binomial multiplied by itself. Examine the square of the binomial $(2x+3)$ and the trinomial that is formed.

$$\begin{aligned}(2x+3)^2 &= (2x+3)(2x+3)\\ &= 4x^2+6x+6x+9\\ &= 4x^2+12x+9\end{aligned}$$

A perfect square trinomial is one that is the product of two identical binomials.

The characteristics are summarized as follows.

- The first term of the trinomial is the square of the first term of the binomial.

$$(2x+3)^2 = \mathbf{4x^2} + 12x + 9$$
$$(2x)^2 = \mathbf{4x^2}$$

- The middle term of the trinomial is twice the product of the two terms of the binomial.

$$(2x+3)^2 = 4x^2 + \mathbf{12x} + 9$$
$$2(2x)(3) = \mathbf{12x}$$

- The sign of the middle term of the trinomial is the same as the sign of the last term of the binomial.

$$(2x \oplus 3)^2 = 4x^2 \oplus 12x + 9$$
$$(2x \ominus 3)^2 = 4x^2 \ominus 12x + 9$$

- The last term of the trinomial is the square of the second term of the binomial.

$$(2x+3)^2 = 4x^2 + 12x + \mathbf{9}$$
$$3^2 = \mathbf{9}$$

In order to recognize a trinomial as the square of a binomial, you have to answer yes to the following:

- Are the first and last terms of the trinomial perfect squares?
- Is the middle term twice the product of the square roots of the first and last term?

Example 11

Is $4a^2 - 12a + 9$ a perfect square trinomial?

Solution:

$\sqrt{4a^2} = 2a$ and $\sqrt{9} = 3$.

The first condition (Are the first and last terms of the trinomial perfect squres?) is statisfied.

$$(2)(2a)(3) = 12a$$

The second condition (Is the middle term twice the product of the square roots of the first and last term?) is satisfied.

Therefore, $4a^2 - 12a + 9$ is a perfect square trinomial.

Because the first and last terms are perfect squares, they will always be positive.

$$(+)^2 = +$$
$$(-)^2 = +$$

Do any four of question 1 and any four of question 2.

1. Determine whether or not each of the following is a perfect square trinomial using a binomial grid or algebra tiles. Binomial grids and algebra tiles are provided in **Appendix B**.

   a. $x^2+8x+16$

   b. x^2-2x+1

   c. x^2+2x-3

   d. $2x^2+12x+3$

   e. x^2+2x-1

   f. $4x^2-4x+1$

   g. $4x^2-12x+9$

   h. $x^2+10x+25$

2. Determine whether or not each of the following is a perfect square trinomial using the algebraic method.

   a. $16x^2-12x+9$

   b. $4x^4+12x^2y^2+9y^4$

   c. $x^2+10x+25$

   d. $25x^2-40x+16$

   e. $x^2+4x+16$

   f. $16x^2-8x+1$

   g. $4x^2-4x+4$

   h. $a^2+2ab+b^2$

For solutions to **Activity 2**, turn to **Appendix A, Topic 2**.

When determining whether a trinomial is a perfect square, ensure the following are satisfied.

- The first and last terms of the trinomial are perfect squares.
- The middle term is twice the product of the square roots of the first and last terms.

Activity 3

Factor a perfect square trinomial.

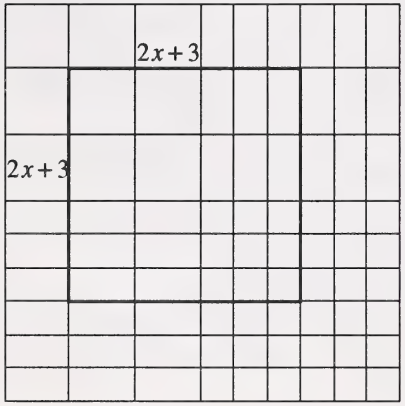
Factoring a perfect square trinomial is just one step more than recognizing a perfect square trinomial.

Example 12

Factor $4x^2+12x+9$ using a binomial grid.

Solution:

Draw the rectangle representing $4x^2+12x+9$ on a binomial grid.



This is a perfect square trinomial with factors $(2x+3)^2$.

Check your answer.

$$(2x+3)(2x+3)=4x^2+6x+6x+9$$
$$=4x^2+12x+9$$

Example 13

Factor $9x^2-30x+25$.

Solution:

negative
$9x^2-30x+25$

$\sqrt{9x^2}=3x$
$\sqrt{25}=5$
$2(3x)(5)=30x$

negative
Therefore, the factors are $(3x-5)^2$.

Check your answer.

$$(3x-5)(3x-5)=9x^2-15x-15x+25$$
$$=9x^2-30+25$$

Do any four of question 1 and any four of question 2.

1. Factor each using a binomial grid or algebra tiles. Binomial grids and algebra tiles are provided in **Appendix B**.

   a. $4x^2 - 4x + 1$

   b. $9x^2 + 24x + 16$

   c. $x^2 - 2x + 1$

   d. $4x^2 + 12x + 9$

   e. $x^2 - 6x + 9$

   f. $9x^2 + 6x + 1$

   g. $4x^2 - 12x + 9$

   h. $x^2 + 4x + 4$

2. Factor each of the following.

   a. $x^2 + 14x + 49$

   b. $16x^2 - 8x + 1$

   c. $144x^2 + 120x + 25$

   d. $243m^2n^2 + 54mn + 3$

   e. $16x^2 - 24xy + 9y^2$

   f. $x^4 + 2x^2y^2 + y^4$

   g. $x^2 + x + \frac{1}{4}$

   h. $x^2 + 18x + 81$

For solutions to **Activity 3,** turn to **Appendix A, Topic 2.**

Activity 4

Recognize the product of conjugate binomials as a special case.

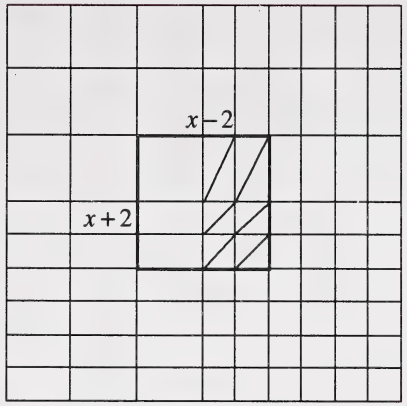
Conjugate binomials are binomials that differ only in the sign of the second term. Some examples of conjugate binomials are $(a+b)$ and $(a-b)$, $(2x+7)$ and $(2x-7)$, and so on.

Look at the product of conjugate binomials.

$$(x+2)(x-2) = x^2 - 2x + 2x - 4$$
$$= x^2 - 4$$

Whenever conjugate binomials are multiplied, the product is a difference of squares.

Examine the product on a binomial grid.



Here you should see that the $2x$ area and the $-2x$ area add to give zero.

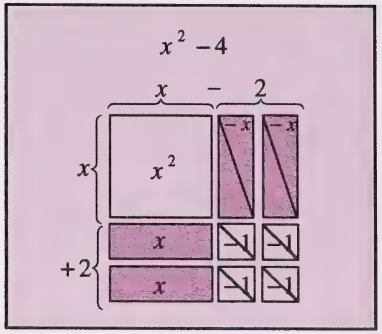
You can recognize the product of conjugate binomials (difference of squares) in the following manner.

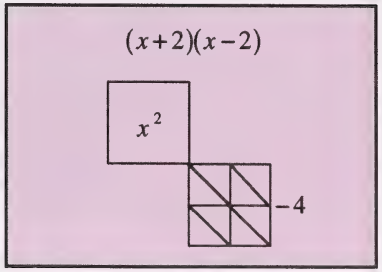
- Are there only two terms, one of which is negative?
- Are the terms perfect squares? (Ignore the negative sign.)

If the answer to both questions is yes, this is a product of conjugate binomials (difference of squares).

The positive x's ( ) and the negative x's ( ) cancel each other out.

$$-2x + 2x = 0$$





Example 14

Is $9x^2 - 16$ a product of conjugate binomials?

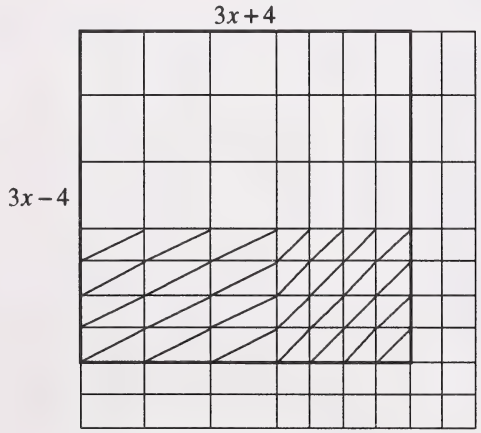
Solution:

There are only two terms, the second is negative.

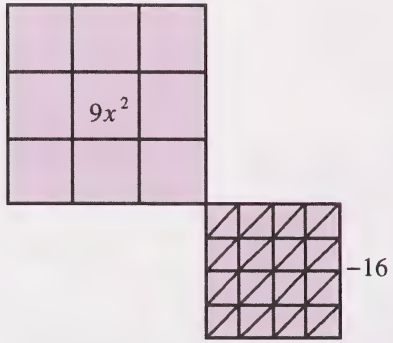
$\sqrt{9x^2} = 3x$ and $\sqrt{16} = 4$.

Therefore, this is the product of conjugate binomials.

You can also solve the question using a binomial grid.



This is equivalent to the following.



This is a difference of squares.

Determine whether or not each of the following is a difference of squares. Do any four of the following.

1. $36a^2 - 25b^4$
2. $y^2 - 16$
3. $36x^2 + 9$
4. $4a^2b^2 - \frac{1}{9}$
5. $100x^2 - 400y^2$
6. $(x+2)^2 - (x+3)^2$
7. $x^2 + 4x + 4$
8. $5x^3 - 4$

For solutions to **Activity 4**, turn to **Appendix A**, **Topic 2**.

Activity 5

Factor a difference of squares binomial.

Once you recognize a difference of squares binomial you need to go one step further in order to factor the binomial.

Take the square root of each term and write the roots as terms in conjugate binomials.

Example 15

Factor $9x^2 - 36$.

Solution:

This is a difference of squares.

$\sqrt{9x^2} = 3x$ and $\sqrt{36} = 6$

Write the roots in the form of conjugate binomials.

$(3x+6)(3x-6)$

Check your answer.

$$(3x+6)(3x-6) = 9x^2 - 18x + 18x - 36$$
$$= 9x^2 - 36$$

Example 16

Factor $64a^4b^6 - \frac{x^6}{4}$.

Solution:

This is a difference of squares.

$$\sqrt{64a^4b^6} = 8a^2b^3 \text{ and } \sqrt{\frac{x^6}{4}} = \frac{x^3}{2}$$

Therefore, the factors are $\left(8a^2b^3 + \frac{x^3}{2}\right)$ and $\left(8a^2b^3 - \frac{x^3}{2}\right)$.

Check your answer.

$$\left(8a^2b^3 + \frac{x^3}{2}\right)\left(8a^2b^3 - \frac{x^3}{2}\right)$$
$$= 64a^4b^6 - 4a^2b^3x^3 + 4a^2b^3x^3 - \frac{x^6}{4}$$
$$= 64a^4b^6 - \frac{x^6}{4}$$

Factor each completely. Do any four. Remember to factor completely.

1. $m^6 - n^2$
2. $x^2 - 16$
3. $x^4 - y^4$
4. $16s^2 - t^2$
5. $y^4 - 1$
6. $\left(x^2 + 1\right)^2 - x^2$
7. $9y^2 - 49b^2$
8. $\frac{x^2}{36} - 1$

For solutions to **Activity 5**, turn to **Appendix A**, **Topic 2**.

Activity 6

Factor polynomials that require a combination of methods in order to be completely factored.

Here are five steps to follow when factoring completely.

Step 1: First factor out the greatest common factor (G.C.F.).

Factor $3x^4 y - 3y$.

$$3x^4 y - 3y = 3y\left(x^4 - 1\right)$$

Step 2: If a factor is a binomial that is a difference of squares, it can be factored.

Factor $3y\left(x^4 - 1\right)$.

$$3y\left(x^4 - 1\right) = 3y\left(x^2 + 1\right)\left(x^2 - 1\right)$$
$$= 3y\left(x^2 + 1\right)(x + 1)(x - 1)$$

Step 3: If a factor is a perfect square trinomial, it can be factored.

Factor $5ax^2+10ax+5a$.

$$5ax^2+10ax+5a=5a\left(x^2+2x+1\right)$$
$$=5a(x+1)^2 \quad \text{(Factor out the G.C.F.)}$$

Step 4: If a factor is a trinomial that is not a perfect square, try to factor it as a product of binomials.

Factor $-2x^2y+2xy+12y$.

$$-2x^2y+2xy+12y=-2y\left(x^2-x-6\right) \quad \text{(Factor out the G.C.F.)}$$
$$=-2y(x-3)(x+2)$$

Step 5: After you factor a polynomial, write all the factors in your final answer. Ensure the factoring is complete. Check your answer by expansion.

Factor $2a^3-50ab^2$ completely.

$$2a\left(a^2-25b^2\right)=2a(a+5b)(a-5b)$$

Example 17

Factor the following.

- $27x^4+18x^2+3$

  Solution:

$$3\left(9x^2+6x^2+1\right)=3\left(3x^2+1\right)\left(3x^2+1\right)$$
$$=3\left(3x^2+1\right)^2$$

- $40x^2+130xy+30y^2$

  Solution:

$$10\left(4x^2+13xy+3y^2\right)=10(4x+y)(x+3y)$$

- $16x^4-8x^2+1$

  Solution:

$$\left(4x^2-1\right)\left(4x^2-1\right)=(2x+1)(2x-1)(2x+1)(2x-1)$$
$$=(2x+1)^2(2x-1)^2$$

Some factoring may be complex. Thus, additional strategies have to be used.

Example 18

Factor the following.

- $35x^3y - 18xy^3$

  Solution:

$$35x^3y - 18xy^3 = xy\left(35x^2 - 18y^2\right)$$
$$= xy\left(\sqrt{35}x + \sqrt{18}y\right)\left(\sqrt{35}x - \sqrt{18}y\right)$$

- $\left(x^2 - xy\right)^2 - 9y^6$

  Solution:

  Let $a = x^2 - xy$.

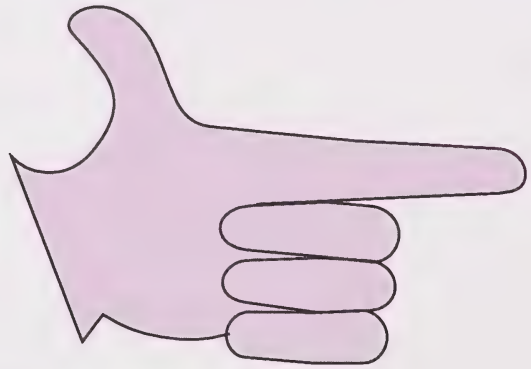
$$\left(x^2 - xy\right)^2 - 9y^6 = a^2 - 9y^6 \quad \text{(Factor.)}$$
$$= \left(a + 3y^3\right)\left(a - 3y^3\right)$$
$$= \left(x^2 - xy + 3y^3\right)\left(x^2 - xy - 3y^3\right) \quad \text{(Substitute } x^2 - xy \text{ for } a.\text{)}$$

- $z\left(b^2 - 25\right) - 6\left(b^2 - 25\right)$

  Solution:

  Notice the binomial is common to both z and -6. Use the common factor once, and multiply by the difference of the other two terms.

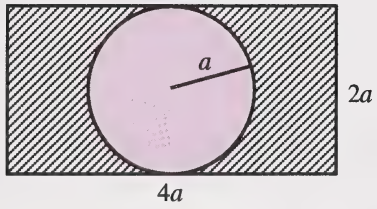
$$z\left(b^2 - 25\right) - 6\left(b^2 - 25\right) = (z - 6)\left(b^2 - 25\right)$$
$$= (z - 6)(b - 5)(b + 5)$$



Often an expression can be simpler to evaluate if it is written in factored form.

Example 19

- Find the expression for the area of the shaded region of the following figure.



Solution:

Area of shaded region = area of rectangle – area of circle

$$= lw - \pi r^2$$
$$= (4a)(2a) - \pi(a)^2$$
$$= 8a^2 - \pi a^2$$

In factored form, $8a^2 - \pi a^2$ is $a^2(8 - \pi)$.

- If $a = 3$, what is the area of the shaded region?

Solution:

Substitute 3 for a in the factored form of the expression.

$$a^2(8 - \pi) = 3^2(8 - 3.14)$$
$$= 9(4.86)$$
$$= 43.74 \text{ units}^2$$

Enter	Display
8	8
−	8
3.14	3.14
×	4.86
9	9
=	43.74

Do any eight of question 1. Solve problems 2 and 3.

1. Factor each completely.

   a. $6x^2y^2 - 9xy^2 + 3xy$

   b. $x^5 - 81x$

   c. $(x+y)^2 - 1$

   d. $-5y^2 - 6y + 11$

   e. $5 - 80a^2$

   f. $16x^2 - 15x - 1$

   g. $xy - x^3y^3$

   h. $(a^2 - ab)^2 - 16b^4$

   i. $12y^2 + 5y - 3$

   j. $64y^2 - 49x^2$

   k. $15a^3b - 11ab^3$

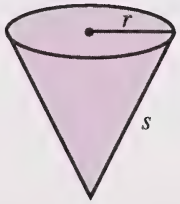
   l. $30x^2 + 10x - 100$

   m. $x^3 + 5x^2 - 36x$

   n. $9y^4 + 6y^2 + 1$

2. The surface area of a cone including the base is given by the following formula:

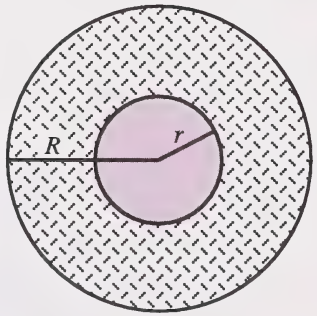
   Surface Area $= \pi rs + \pi r^2$



   a. Factor the expression.

   b. Find the surface area if $\pi = 3.14$, $r = 3$ cm, and $s = 5$ cm.

3. The shaded area A is given by the formula $A = \pi R^2 - \pi r^2$.



   a. Simplify the formula by factoring.

   b. Calculate the shaded area if $r = 7$ cm, $R = 12$ cm, and $\pi = 3.14$.

For solutions to **Activity 6**, turn to **Appendix A**, **Topic 2**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

If you are still having problems factoring, look at a technique developed by a student that provides a structured, systematic method of factoring trinomials and special case binomials. It is called, the L method. You will be led, step by step, through the method using various examples.

Factor $x^2+2x-15$.

Step 1: Draw a big L.

Multiply the first and the last terms.

$$x^2+2x-15$$

↑ first term ↑ last term

$$(x^2)(-15)=-15x^2$$

Write this term to the left of the L.

$-15x^2$ |

Step 2: Write the middle term below the L.

$-15x^2$ |
$2x$

Step 3: Find two terms whose product is $-15x^2$ (the term on the left) and whose sum is $2x$ (the term at the bottom).

product	
$-15x^2$	$5x$
	$-3x$
	$2x$
	sum

The idea looks like this:

$$\begin{array}{r|l} -15x^2 & \leftarrow \times \quad 5x \\ & \qquad -3x \\ \hline & \qquad 2x \end{array}$$

The terms multiplied give the value on the left.

$$\begin{array}{r|l} -15x^2 & + \quad 5x \\ & \qquad -3x \\ & \qquad \downarrow \\ \hline & \qquad 2x \end{array}$$

The terms added give the value on the bottom.

The two terms you find replace the middle term of the original trinomial.

Step 4: Write the first term of the original trinomial, followed by the two terms in the middle of your L, followed by the last term of the original trinomial.

$$x^2 + 5x - 3x - 15$$

Step 5: Group the terms and factor.

$$\underbrace{x^2 + 5x}\,\underbrace{-3x - 15} = x(x+5) - 3(x+5)$$
$$= (x-3)(x+5)$$

Check your factoring using expansion.

$$(x-3)(x+5) = x^2 + 5x - 3x - 15$$
$$= x^2 + 2x - 15$$

Therefore, the factors are $(x-3)(x+5)$.

Here is an example with less detail.

Factor $x^2 + 7x + 12$.

Step 1: $(x^2)(12) = 12x^2$

$$\begin{array}{r|l} 12x^2 & \\ \hline & \end{array}$$

Step 2:

$$\begin{array}{r|l} 12x^2 & \\ \hline & 7x \end{array}$$

Step 3: There are three possibilities for $12x^2$.

$(12x)(1x)$
$(6x)(2x)$
$(4x)(3x)$

All three possibilities are shown here.

$$\underbrace{\begin{array}{r|l} 12x^2 & \mathbf{12x} \\ & \mathbf{1x} \\ \hline & 7x \end{array}}$$

These two do not work.

$$\underbrace{\begin{array}{r|l} 12x^2 & \mathbf{6x} \\ & \mathbf{1x} \\ \hline & 7x \end{array}}$$

$$\underbrace{\begin{array}{r|l} 12x^2 & 4x \\ & 3x \\ \hline & 7x \end{array}}$$

This one does work.

Step 4: $x^2+4x+3x+12$

Step 5: Group the terms and factor.

$$(x^2+4x)+(3x+12)=x(x+4)+3(x+4)$$
$$=(x+3)(x+4)$$

Check your answer.

$$(x+3)(x+4)=x^2+4x+3x+12$$
$$=x^2+7x+12$$

Now work through this example.

Example 20

Factor $2x^2+7x-15$.

Solution:

Step 1: $(2x^2)(-15)=-30x^2$

$$-30x^2 \mid \quad$$

Step 2:

$$-30x^2 \mid \quad$$
$$7x$$

Step 3: The possibilities are as follows:

$(-15x)(2x)$ $(15x)(-2x)$
$(-5x)(6x)$ $(5x)(-6x)$
$(-3x)(10x)$ $(3x)(-10x)$

The one required is $(-3x)(10x)$ because $-3x+10x=7x$.

$$-30x^2 \mid \begin{matrix} 10x \\ -3x \end{matrix}$$
$$7x$$

Step 4: $2x^2+10x-3x-15=2x(x+5)-3(x+5)$
$$=(2x-3)(x+5)$$

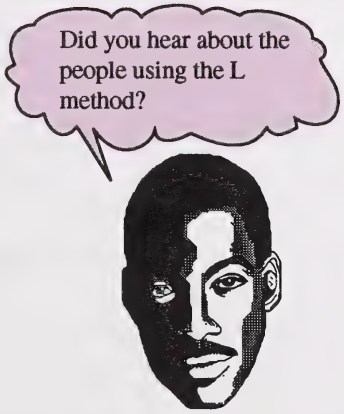
Check your answer.

$$(2x-3)(x+5)=2x^2+10x-3x-15$$
$$=2x^2+7x-15$$

Factor using the L method. Try as many of the following as you feel necessary.

1. $x^2+3x-10$
2. $x^2-8x+12$
3. $x^2+14x+49$
4. $4x^2+20x+25$
5. $4x^2-12x+9$
6. x^2-6x+8
7. x^2+4x-5
8. $3x^2+16x-35$

For solutions to **Extra Help**, turn to **Appendix A**, **Topic 2**.



Extensions

Factoring has many uses, especially in mathematics and fields strongly dependent on mathematics such as engineering and science.

You can simplify complicated expressions using factoring.

Simplify $\frac{x^2-4}{x+2}$.

If you have common factors in the numerator and the denominator, you can divide out the common factors.

Since $x^2-4=(x+2)(x-2)$, you can rewrite the equations as follows:

$$\frac{x^2-4}{x+2}=\frac{(x+2)(x-2)}{(x+2)}$$

Divide out the common factor.

$$\frac{\cancel{(x+2)}^{1}(x-2)}{\cancel{(x+2)}_{1}}=x-2$$

Therefore, $\frac{x^2-4}{x+2}=x-2$.

When adding or subtracting complex fractions, use factoring to help find the lowest common denominator.

Add $\frac{1}{x^2-3x}+\frac{1}{x-3}$.

Factor the denominators to help find the lowest common denominator. $x-3$ is not factorable.

$$x^2-3x=x(x-3)$$

Now rewrite the expression with the common denominator.

$$\frac{1}{x(x-3)}+\frac{x}{x(x-3)}=\frac{1+x}{x(x-3)}$$
$$=\frac{1+x}{x^2-3x}$$

There are some interesting number curiosities that utilize factoring.

For example, multiply 49×51 mentally. (Do not use paper and pencil.)

Notice that $49=50-1$ and $51=50+1$.

Therefore, $49\times51=(50-1)(50+1)$.

This is a difference of squares: $(50-1)(50+1)=50^2-1^2$
$$=2500-1$$
$$=2499$$

Try as many of the following as you feel necessary.

1. Simplify each of the following.

   a. $\frac{x^2-2x+1}{x-1}$ b. $\frac{x^2-4}{x^2+4x+4}$

   c. $\frac{x^2-5x+6}{x^2+2x-15}$ d. $\frac{y^2-1}{y-1}$

2. Add or subtract, and simplify completely.

   a. $\frac{4}{xy}+\frac{5}{x}$ b. $\frac{2}{x^2-1}+\frac{3}{x^2-2x+1}$

   c. $\frac{x^2-9}{x^2-5x+6}-\frac{3}{x-2}$ d. $\frac{2x}{4x^2+8x}-\frac{x}{2x+4}$

3. Mentally multiply the following.

   a. 99×101

   b. 499×501

For solutions to **Extensions**, turn to **Appendix A, Topic 2**.

Topic 3 Solving Equations by Using Factoring

Introduction

In this topic you will solve problems that require factoring. For example, if a projectile is fired upwards with a velocity of 1540 m/s, how long will it take to reach a height of 12 000 m?

You will learn to solve this particular problem and many more in this topic.

What Lies Ahead

Throughout the topic you will learn to

1. find the rational zeros of an integral quadratic polynomial in one variable by factoring
2. solve quadratic equations with integral coefficients in one variable by factoring
3. solve problems that can be represented by factorable quadratic equations

Now that you know what to expect, turn the page to begin your study of solving equations by using factoring.

Exploring Topic 3

Activity 1

Find the rational zeros of an integral quadratic polynomial in one variable by factoring.

The zero product property states that if $(a)(b)=0$, then $a=0$ and/or $b=0$.

More simply, if numbers are multiplied together and the product is zero, at least one of the factors must be equal to zero.

Here are some examples to illustrate this property.

- $(2)(5)(0)(3)=0$ (since one factor is zero)
- $(4)(2b)(0)(7)(y)=0$ (since one factor is zero)
- $(3)(2+x)(0)=0$ (since one factor is zero)

In order to solve an equation like $x^2-5x-6=0$, do the following.

- Factor the equation.
- Set each factor equal to zero.
- Solve for the variable.

Example 1

Solve $x^2+x-6=0$

Solution:

Factor the equation.

$$(x+3)(x-2)=0$$

Set each factor to zero and solve.

$$x+3=0 \quad \text{or} \quad x-2=0$$
$$x=-3 \qquad\qquad x=2$$

Therefore, the solution to the equation is $x=2$ and $x=-3$.

Example 2

For what values of x is $2x^3 - 10x^2 - 28x$ equal to zero?

Solution:

Let $2x^3 - 10x^2 - 28x = 0$. Factor the equation.

$$(2x)(x^2 - 5x - 14) = 0$$
$$(2x)(x-7)(x+2) = 0$$

Set each factor to zero and solve.

$2x = 0$ $\quad$ $x - 7 = 0$ $\quad$ $x + 2 = 0$

$x = 0$ $\quad$ $x = 7$ $\quad$ $x = -2$

Check your answer.

When $x = 0$:

LS	RS
$2x^3 - 10x^2 - 28x$	0
$2(0)^3 - 10(0)^2 - 28(0)$	0
$2(0) - 10(0) - 28(0)$	0
$0 - 0 - 0$	0
0	0
LS =	RS

When $x = 7$:

LS	RS
$2x^3 - 10x^2 - 28x$	0
$2(7)^3 - 10(7)^2 - 28(7)$	0
$2(343) - 10(49) - 28(7)$	0
$686 - 490 - 196$	0
0	0
LS =	RS

When $x = -2$:

LS	RS
$2x^3 - 10x^2 - 28x$	0
$2(-2)^3 - 10(-2)^2 - 28(-2)$	0
$2(-8) - 10(4) - 28(-2)$	0
$-16 - 40 + 56$	0
0	0
LS =	RS

Therefore, the polynomial is equal to zero when $x = 0$, $x = 7$, or $x = -2$.

The real zeros of the polynomial are $x = 0$, $x = 7$, and $x = -2$.

The solutions to the equations are called the **real zeros** of the polynomial.

Example 3

Find the real zeros of $(2x^2-2)(x+2)$.

Solution:

Let $(2x^2-2)(x+2)=0$. Factor the polynomial.

$$2(x^2-1)(x+2)=0$$
$$2(x+1)(x-1)(x+2)=0$$

Set each of the factors to zero and solve.

$x+1=0$ or $x-1=0$ or $x+2=0$

$x=-1$ $\quad x=1$ $\quad x=-2$

Check your answer.

When $x=-1$:

LS	RS
$(2x^2-2)(x+2)$	0
$[2(-1)^2-2][(-1)+2]$	0
$[2(1)-2][(-1)+2]$	0
$(2-2)(-1+2)$	0
$(0)(1)$	0
0	0
LS =	RS

When $x=1$:

LS	RS
$(2x^2-2)(x+2)$	0
$[2(1)^2-2][(1)+2]$	0
$[2(1)-2][(1)+2]$	0
$(2-2)(1+2)$	0
$(0)(3)$	0
0	0
LS =	RS

When $x=-2$:

LS	RS
$(2x^2-2)(x+2)$	0
$[2(-2)^2-2][(-2)+2]$	0
$[2(4)-2][(-2)+2]$	0
$(8-2)(-2+2)$	0
$(6)(0)$	0
0	0
LS =	RS

Therefore, the real zeros are $x=-1$, $x=1$, and $x=-2$.

$$(-1)^2=(-1)(-1)$$
$$=1$$

Determine the real zeros by factoring. Do any six of the following.

1. x^2+2x-3
2. $25k^2-49$
3. $2y^3-50y$
4. $3x^2-x-2$
5. $2k^2-5k-3$
6. $x^2+7x-18$
7. $3k^2+k-30$
8. $6y^2-13y+6$
9. $2y^2+5y+2$
10. x^2-2x+1

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 3**.

Activity 2

Solve quadratic equations with integral coefficients in one variable by factoring.

In the previous activity you learned to find the real zeros of polynomials. Solving quadratic equations relies on your ability to find the real zeros of polynomials.

Example 4

Solve for x in $x^2-6x=-5$.

Solution:

Add 5 to both sides to make the equation equal to zero.

$$x^2-6x+5=-5+5$$
$$x^2-6x+5=0$$

Factor the polynomial.

$$(x-5)(x-1)=0$$

Let each factor equal zero and solve for x.

$$x-5=0 \text{ or } x-1=0$$
$$x=5 \qquad x=1$$

Check your answer.

When $x = 5$:

LS		RS
$x^2 - 6x$		-5
$(5)^2 - 6(5)$		-5
$25 - 30$		-5
-5		-5
LS	=	RS

When $x = 1$:

LS		RS
$x^2 - 6x$		-5
$(1)^2 - 6(1)$		-5
$1 - 6$		-5
-5		-5
LS	=	RS

Therefore, the solutions to $x^2 - 6x = -5$ are $x = 5$ and $x = 1$.

The solutions to an equation are commonly referred to as the roots of the equation. The roots of the equation in Example 4 are 5 and 1.

The steps to solving quadratic equations are as follows.

Step 1: Rewrite the equation in the form $ax^2 + bx + c = 0$ so that the right-hand side equals zero.

Step 2: Factor the left-hand side if possible.

Step 3: Set each of the factors equal to zero and solve for the variable.

Step 4: Check by substituting into the original equation.

Example 5

Find the roots of $2x^2 + 9x = 5$.

Solution:

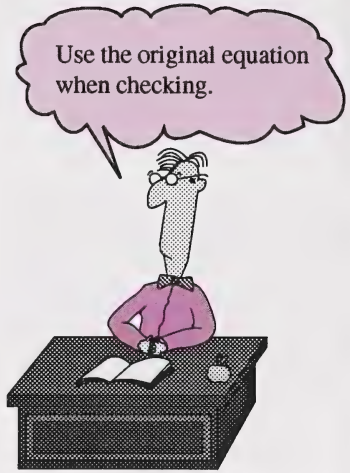
Step 1: $2x^2 + 9x - 5 = 0$

Step 2: $(2x - 1)(x + 5) = 0$

Step 3: $2x - 1 = 0$ or $x + 5 = 0$

$2x = 1$ $\qquad x = -5$

$x = \frac{1}{2}$



Step 4: Check

When $x=\frac{1}{2}$:

LS	RS
$2x^2+9x$	5
$2\left(\frac{1}{2}\right)^2+9\left(\frac{1}{2}\right)$	5
$2\left(\frac{1}{4}\right)+9\left(\frac{1}{2}\right)$	5
$\frac{1}{2}+\frac{9}{2}$	5
$\frac{10}{2}$	5
5	5
LS =	RS

When $x=5$:

LS	RS
$2x^2+9x$	5
$2(-5)^2+9(-5)$	5
$2(25)+(-45)$	5
$50-45$	5
5	5
LS =	RS

Therefore, the roots of the equation are $x=\frac{1}{2}$ and $x=-5$.

For additional work with factoring and solving quadratic equations, use the computer program titled *Solving Quadratic Equations*[1]. Do as many questions as you feel are necessary.

Find the real roots for each. Do any six of the following.

1. $3x^3+6x^2=9x$
2. $x^2=4x+45$
3. $18=x^2+3x$
4. $x^2+14=-9x$
5. $x^4+7x^3+12x^2=0$
6. $x^2+6x=-9$
7. $x^2+25=-10x$
8. $2x^2=x+3$
9. $2x^2+7x=4$
10. $x^3=49x$

For solutions to **Activity 2**, turn to **Appendix A**, **Topic 3**.

[1] Solving Quadratic Equations is a title of *Microcomputer Workshops*.

Activity 3

Solve problems that can be represented by factorable quadratic equations.

A common physics problem involves finding the time it takes for a projectile to reach a certain height. When a word problem is translated into an algebraic expression, the result may be a quadratic equation.

Example 6

Anti-aircraft fire involves the firing of a timed explosive shell that will explode at a set time after being fired. The shrapnel from the exploding shell could damage an aircraft if the shell explodes close enough. The height of a targeted aircraft is the height where the anti-aircraft shell is set to explode.

The formula for determining the height of the projectile is given by $h = v_i t + \frac{1}{2}at^2$, where

h = height in metres

v_i = initial velocity of the projectile

a = the acceleration due to gravity $\left(-10 \text{ m/s}^2\right)$

t = time in seconds

An aircraft is flying at a height of 7000 m. What time (in seconds) must the anti-aircraft shells be set so they explode at the height of the aircraft? The shells are fired with an initial velocity of 1425 m/s.

Solution:

Substitute the values for h, v_i, and a into the equation and simplify. (The units are omitted here to make the equation easier to read.)

$$h = v_i t + \frac{1}{2}at^2$$

$$7000 = (1425)t + \frac{1}{2}(-10)t^2$$

$$7000 = 1425t - 5t^2$$

This is a quadratic equation that can be solved.

Step 1: Rewrite the equation so that the right side is equal to zero.

$$5t^2 - 1425t + 7000 = 0$$

Step 2: Factor.

$$5\left(t^2 - 285t + 1400\right) = 0$$

$$5(t-5)(t-280) = 0$$

Since the projectile is moving away from the earth, a is negative because gravity causes it to slow down. When an object falls toward the earth, a is positive since the object speeds up.

Step 3: Set the factors to zero and solve.

$t-5=0$ or $t-280=0$

$t=5$ $\qquad$ $t=280$

Step 4: Check your answer:

When $t=5$:

LS	RS
7000	$1425t-5t^2$
7000	$1425(5)-5(5)^2$
7000	$7125-125$
7000	7000

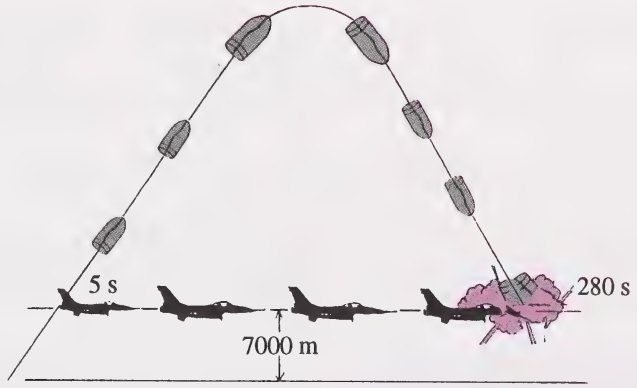
LS = RS

When $t=280$:

LS	RS
7000	$1425t-5t^2$
7000	$1425(280)-5(280)^2$
7000	$399\,000-392\,000$
7000	7000

LS = RS

There are two solutions (5 s and 280 s) because the shell will be at 7000 m twice, once on the way up and once on the way down. Look at the following diagram.



Therefore, the shells must be set to explode 5 s or 280 s after they are fired. In reality, the shorter time will be used even though the diagram shows it exploding on the way down at 280s.

Example 7

Two numbers differ by 3. The sum of their squares is 65. Find the numbers.

Solution:

Let x be the larger number. The square of x is x^2.

Let $x-3$ be the smaller numer. The square of $(x-3)$ is $(x-3)^2$.

The statement **the sum of their squares is 65** translates into the expression

$x^2+(x-3)^2=65$.

Solve the equation for the roots.

Step 1: $x^2+x^2-6x+9=65$

$2x^2-6x-56=0$

Step 2: Factor.

$2\left(x^2-3x-28\right)=0$

$2(x-7)(x+4)=0$

Step 3: $x-7=0$ or $x+4=0$

$x=7$ $\qquad$ $x=-4$

Step 4: Check your answer.

When $x=7$:

LS	RS
$x^2+(x-3)^2$	65
$7^2+(7-3)^2$	65
7^2+4^2	65
$49+16$	65
65	65
LS =	RS

$x=-4$

LS	RS
$x^2+(x-3)^2$	65
$(-4)^2+(-4-3)^2$	65
$(-4)^2+(-7)^2$	65
$16+49$	65
65	65
LS =	RS

If you check the original problem for both these values, you get the following.

$x=7$	$x=-4$
$x-3=4$	$x-3=-7$

The two numbers are 7 and 4 or -4 and -7.

Both conditions of the original equation are satisfied. The numbers differ by three and the sum of the squares is 65.

$(x-3)^2=(x-3)(x-3)$

$=x^2-6x+9$

Example 8

If you subtract five from twice a certain number and square the result, you get 121. What is the number?

Solution:

Let x be the number.
Subtract 5 from twice the number $= (2x-5)$.
Then square the result.

$$(2x-5)^2 = 121$$
$$4x^2 - 20x + 25 = 121$$
$$4x^2 - 20x - 96 = 0$$
$$4\left(x^2 - 5x - 24\right) = 0$$
$$4(x-8)(x+3) = 0$$
$$x-8=0 \text{ or } x+3=0$$
$$x = 8 \qquad x = -3$$

Check your answer.

When $x = 8$:

LS	RS
$(2x-5)^2$	121
$[2(8)-5]^2$	121
$(16-5)^2$	121
11^2	121
121	121
LS =	RS

When $x = -3$:

LS	RS
$(2x-5)^2$	121
$[2(-3)-5]^2$	121
$(-6-5)^2$	121
$(-11)^2$	121
121	121
LS =	RS

Therefore, the number is 8 or -3. Both numbers are correct.

Sometimes students have trouble interpreting word problems. If you are having trouble, be sure to go over the examples very carefully. Try to follow the patterns in your own problem solving. It is a very important skill to master.

Solve the following problems, Do any four.

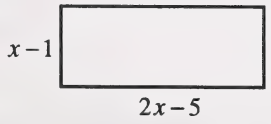
1. Two numbers differ by 7. The sum of the squares of the numbers is 85. Find the numbers.

2. Two numbers added together equal 20. The sum of the squares of the numbers is 298. Find the numbers.

3. A projectile is fired upwards with a velocity of 1540 m/s. Determine the time required for the object to reach a height of 12 000 m.

   Use the formula $h = v_i t + \frac{1}{2}at^2$ and let $a = -10$.

4. The sides of a rectangle are given as in the diagram. If the area of the rectangle is $54\ m^2$, determine the lengths of the sides.



5. A triangle has a base that is 2 cm longer than the height. If the area of the triangle is $24\ cm^2$, find the base and the height of the triangle.

6. Multiply a number by three; then, subtract two from the result. If you square this result, you will get 100. Find the two numbers that satisfy these conditions.

For solutions to **Activity 3**, turn to **Appendix A**, **Topic 3**.

A problem-solving procedure is listed as follows.

- Understand the problem.
- Develop a plan.
- Carry out the plan.
- Look back.

The area of a triangle is as follows.

$$A = \frac{1}{2}bh$$

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Sometimes students can find the roots of a mathematical equation but cannot solve word problems that involve the very same mathematical equation. Why?

Problem-solving skills can be developed through a long-term exposure to word problems. Problem-solving techniques can be learned through practice, and a desire to succeed in solving word problems. In this section you will see a new perspective on word problems which may be helpful in solving word problems.

Problem: Two numbers differ by 5. The difference of the squares of these numbers is 85. Find the numbers.

Solution: There are two restricting conditions in this problem. The first condition is that the numbers differ by 5. There are an infinite number of pairs of numbers that satisfy this one condition. Take any number and subtract 5 from it. The result is the other number.

For example, use 20.

Subtract five from the number $(20-5=15)$.

The result is a pair of numbers (20, 15) that differs by five.

Since there are an infinite number of pairs that satisfy the problem, use algebra to write the expression for these numbers.

$$x-y=5$$

Here, x is the larger number and y is the smaller number.

This algebraic expression can be written in different ways by rearranging the equation.

	Check using $x=20,\ y=15$.
If $x-y=5$,	$20-15=5$ ✓
then $x=y+5$,	$20=15+5$ ✓
or $y=x-5$,	$15=20-5$ ✓
or $x-y-5=0$.	$20-15-5=0$ ✓

You could have selected any of these equation. It depends on how you interpreted the first condition.

Now you must account for the second condition. **The difference of the squares of these numbers is 85.**

Look at the first pair that was picked, 20 and 15. The squares of these numbers are $20^2=20\times20=400$ and $15^2=15\times15=225$.
The difference of the squares is $400-225=175$.
The second condition is not satisfied.

You could take all the pairs of numbers that satisfy the first condition and check to see if the second condition is satisfied. However, this is very tedious and time-consuming. Use algebra again.

$$x^2 - y^2 = 85$$

Combine both equations in order to get one equation that satisfies both conditions.

Since $x = y + 5$, then $x^2 = (y+5)^2$. Substitute $(y+5)^2$ for x^2 in $x^2 - y^2 = 85$ to get $(y+5)^2 - y^2 = 85$.

Use your mathematical skills to solve the equation.

$$\begin{aligned}(y+5)^2 - y^2 &= 85\\ y^2 + 10y + 25 - y^2 &= 85\\ 10y + 25 &= 85\\ 10y &= 85 - 25\\ 10y &= 60\\ y &= \frac{60}{10}\\ &= 6\end{aligned}$$

The number is 6. Therefore, the larger number must be $6 + 5 = 11$. The two numbers are 11 and 6.

Check the conditions.

Is $11 - 6 = 5$? ✓

Is $11^2 - 6^2 = 85$?

$121 - 36 = 85$ ✓

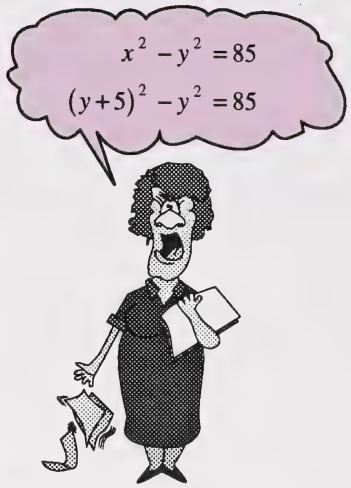
Was this the only way to solve the problem? No, there are more ways to set up the equations and the combined equation.

For example, you may substitute $(x-5)$ for y in $x^2 - y^2 = 85$ to get the equation $x^2 - (x-5)^2 = 85$.

$$\begin{aligned}x^2 - (x-5)^2 &= 85\\ x^2 - (x^2 - 10x + 25) &= 85\\ x^2 - x^2 + 10x - 25 &= 85\\ 10x - 25 &= 85\\ 10x &= 110\\ x &= 11\end{aligned}$$

Thus, $y = 11 - 5 = 6$.

The numbers are still 11 and 6.



Look at a very similar problem.

Example 9

Two numbers differ by 5. The sum of the squares of these numbers is 73. Find the numbers.

Solution:

Proceed exactly as in the previous example.

The first condition is $x-y=5$ or $x=y+5$

The second condition is $x^2+y^2=73$.

Substitute $(y+5)$ for x in $x^2+y^2=73$ to get $(y+5)^2+y^2=73$.

Solve the equation.

$$y^2+10y+25+y^2=73$$
$$2y^2+10y+25=73$$

Now use your skills in factoring to solve this equation.

$$2y^2+10y-48=0$$
$$2(y^2+5y-24)=0$$
$$2(y+8)(y-3)=0$$
$$y+8=0 \quad \text{or} \quad y-3=0$$
$$y=-8 \qquad y=3$$

Solve for x when $y=3$.

$$x-y=5$$
$$x-3=5$$
$$x=8$$

Check your answer.

$8-3=5$ ✓

$8^2+3^2=64+9$

$=73$ ✓

One pair of numbers is 3 and 8.

Solve for the other value of x when $y=-8$.

$$x-(-8)=5$$
$$x+8=5$$
$$x=-3$$

Check your answer.

$-3-(-8)=5$ ✓

$(-3)^2+(-8)^2=9+64$

$=73$ ✓

Another pair of numbers that satisfies the conditions is -3 and -8.

Try some of these problems yourself.

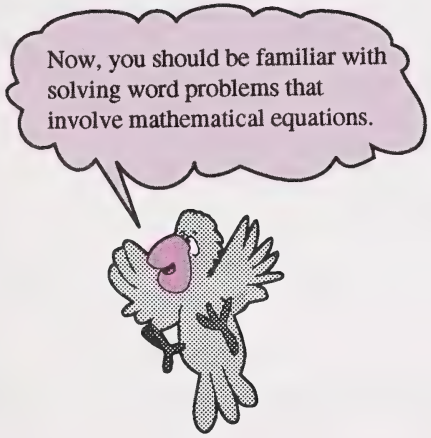
1. The difference between two numbers is 5. The difference of the squares of these numbers is 45. Find the numbers.

2. The difference between two numbers is 5. The sum of the squares of these numbers is 97. Find the numbers.

3. The sum of the two numbers is 10. The sum of the squares of the two numbers is 68. Find the numbers.

4. The sum of two numbers is 15. The difference of the squares of the two numbers is 45. Find the numbers.

For solutions to **Extra Help**, turn to **Appendix A, Topic 3.**



Extensions

Solving Quadratic Inequalities

Consider this example.

Example 10

Find the value for x that makes the following inequality true. Graph the values on a number line.

$$x^2 - 6x + 8 > 0$$

Solution:

First, factor the left-hand side to determine the zero.

$$x^2 - 6x + 8 > 0$$
$$(x-4)(x-2) > 0$$

The zeros $x = 2$ and $x = 4$ are the values of x that will make the quadratic equal to zero. Therefore, you have to determine the range of x-values that will make the original question true.

If $x > 4$, the inequality is true. For example, try $x = 5$. Then

$$\begin{aligned} x^2 - 6x + 8 &= (5)^2 - 6(5) + 8 \\ &= 25 - 30 + 8 \\ &= 3 \quad \text{(greater than zero)} \end{aligned}$$

If $x<2$, the inequality is also true. For example try $x=1$. Then

$$\begin{aligned} x^2-6x+8 &= 1^2-6(1)+8 \\ &= 1-6+8 \\ &= 3 \quad \text{(greater than zero)} \end{aligned}$$

If $x>2$ and $x<4$, then the inequality is false. For example, if you try $x=3$, then

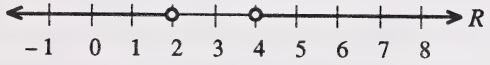
$$\begin{aligned} x^2-6x+8 &= (3)^2-6(3)+8 \\ &= 9-18+8 \\ &= -1 \quad \text{(less than zero)} \end{aligned}$$

Algebraically write the solutions as follows:

$x<2$ or $x>4$, $x \in R$.

This is read as ***x* is less than 2 or *x* is greater than 4**; x is an element of the set of real numbers.

The graph of the solution is as follows.



Through this example, you can see how to solve inequalities.

- Factor the quadratic.
- From the factors, determine the solution range.
- Draw the graph.

Try some of these questions. Solve and write the solution algebraically; and draw the graph of the solution.

1. $x^2-2x-8<0$

2. $x^2+x-12>0$

3. $x^2-6x-7<0$

4. $x^2-3x-10>0$

For solutions to **Extensions**, turn to **Appendix A, Topic 3**.

Unit Summary

What You Have Learned

You have learned the following.

- to factor a polynomial that has a common monomial factor

  Factor $3x^2y - 18xy + 9y$.

  $$3x^2y - 18xy + 9y = 3y(x^2 - 6x + 3)$$

- to factor a polynomial that has a common binomial factor

  Factor $3a - bx + 3b - ax$.

  $$3a + 3b - ax - bx = 3(a+b) - x(a+b)$$
  $$= (a+b)(3-x)$$

- to factor a trinomial of the form $ax^2 + bx + c$, where $a, b, c \in I$

  Factor $2x^2 - 5x - 12$.

  $$2x^2 - 5x - 12$$
  $$(2x+3)(x-4)$$

- to recognize the expansion of the square of a binomial as a special case

  Is $x^2 + 4x + 4$ a perfect square trinomial?

  Yes, because $x^2 + 4x + 4 = (x+2)^2$.

- to factor a perfect square trinomial

  Factor $x^2 - 6x + 9$.

  $$x^2 - 6x + 9 = (x-3)^2$$

Unit Summary

- to recognize the product of conjugate binomials as a special case

  Is $x^2 - 16$ a product of conjugate binomials?

  Yes, since $x^2 - 16 = (x+4)(x-4)$.

- to factor a difference of squares binomial

  Factor $4x^4 - 25y^6$.

  $$\left(4x^4 - 25y^6\right) = \left(2x^2 + 5y^3\right)\left(2x^2 - 5y^3\right)$$

- to factor polynomials that require a combination of methods in order to be fully factored

  Factor $2x^3y^2 - 2xy^2$.

  $$\begin{aligned} 2x^3y^2 - 2xy^2 &= 2xy^2\left(x^2 - 1\right) \\ &= 2xy^2(x+1)(x-1) \end{aligned}$$

- to find the zeros of an integral quadratic polynomial in one variable by factoring

  Find the rational zeros of $2x^2 - 13x - 7$.

  Let $2x^2 - 13x - 7 = 0$, and factor the expression.

  $$(2x+1)(x-7) = 0.$$

  Set each factor to zero and solve.

  $$\begin{aligned} 2x+1 = 0 \quad &\text{or} \quad x-7 = 0 \\ 2x = -1 \quad &\qquad\quad x = 7 \\ x = -\frac{1}{2} \quad & \end{aligned}$$

  The zeros of $2x^2 - 13x - 7$ are $-\frac{1}{2}$ and 7.

Unit Summary

- to solve quadratic equations with integral coefficients in one variable by factoring

Solve for x in the equation $3x^2 - 4x - 4 = 0$.

$$3x^2 - 4x - 4 = 0$$
$$(3x+2)(x-2) = 0$$

$$3x+2=0 \quad \text{or} \quad x-2=0$$
$$3x=-2 \qquad\qquad x=2$$
$$x=-\frac{2}{3}$$

The roots of $3x^2 - 4x - 4 = 0$ are $-\frac{2}{3}$ and 2.

- to solve problems that can be represented by factorable quadratic equations

Two numbers differ by 4. The sum of their squares is 106. Find the numbers.

Let x be the smaller number.
Let $x+4$ be the larger number.

$$x^2 + (x+4)^2 = 106$$
$$x^2 + x^2 + 8x + 16 = 106$$
$$2x^2 + 8x - 90 = 0$$
$$2\left(x^2 + 4x - 45\right) = 0$$
$$2(x+9)(x-5) = 0$$

$$x+9=0 \quad \text{or} \quad x-5=0$$
$$x=-9 \qquad\qquad x=5$$

The numbers are -9 and -5 or 5 and 9.

You are now ready to

complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Removing Common Factors

Topic 2 Factoring

Topic 3 Solving Equations by Using Factoring

Appendix B
Grids and Tiles

Binomial Grids

Algebra Tiles

Appendix A
Solutions

Review

1. $3x^2 - 2y^2 = 3(3)^2 - 2(-4)^2$
$= 3(9) - 2(16)$
$= 27 - 32$
$= -5$

2. a. $(2a^2b^{-3})(-3ab^2) = (2)(-3)(a^{2+1})(b^{-3+2})$
$= -6a^3b^{-1}$
$= \frac{-6a^3}{b}$

b. $\frac{4x^2y^5}{-2x^4y^2} = \frac{4}{-2}(x^{2-4})(y^{5-2})$
$= -2x^{-2}y^3$
$= \frac{-2y^3}{x^2}$

c. $(-2x^2y^{-3})^2 = (-2x^2y^{-3})(-2x^2y^{-3})$
$= [-2\times(-2)](x^{2+2})(y^{-3+(-3)})$
$= 4x^4y^{-6}$
$= \frac{4x^4}{y^6}$

d. $(-3x^{-2}y)^0 = 1$ (Any number to the zero exponent is equal to one.)

e. $2x^{-3} = \frac{2}{x^3}$

f. $\frac{9}{3x^{-2}} = 3x^2$

3. a. $2x^2(y^3 - 3x) = 2x^2(y^3) - 2x^2(3x)$
$= 2x^2y^3 - 6x^3$

b. $(x-3)(x+2) = x^2 + 2x - 3x - 6$
$= x^2 - x - 6$

c. $(x-2)^2 = (x-2)(x-2)$

$= x^2 - 2x - 2x + 4$

$= x^2 - 4x + 4$

d. $(3x-y)(2y-x) = 6xy - 3x^2 - 2y^2 + xy$

$= 7xy - 3x^2 - 2y^2$

$= -3x^2 + 7xy - 2y^2$

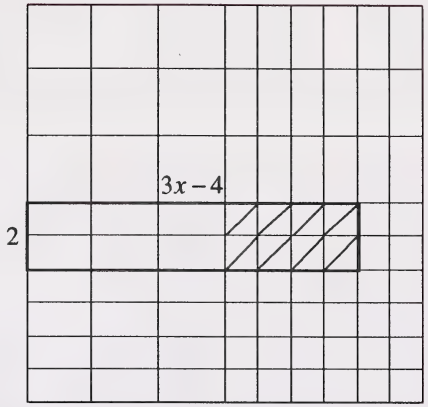
4. a. $3xy^2$ b. x^2yz^2

Exploring Topic 1

Activity 1

Factor a polynomial that has a common monomial factor.

1. a.



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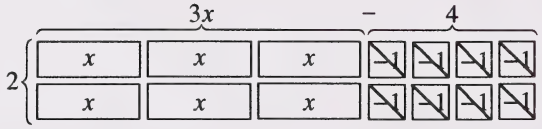
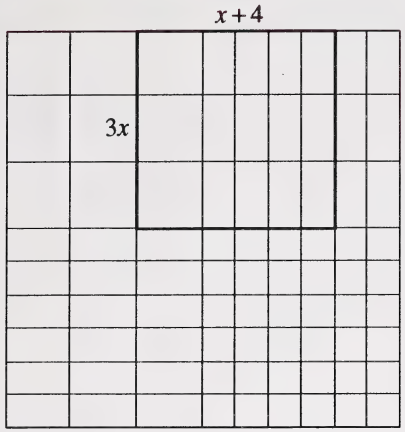


$\therefore 6x - 8 = 2(3x - 4)$

b.



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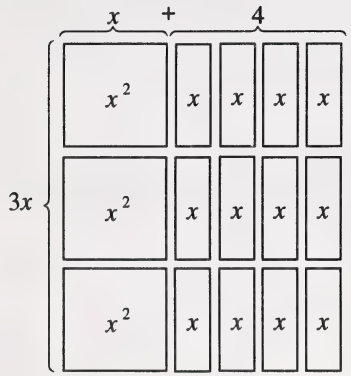
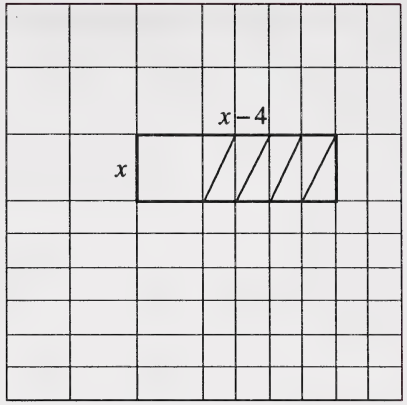
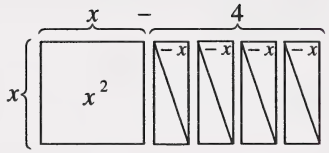


$\therefore 3x^2 + 12x = 3x(x+4)$

c.

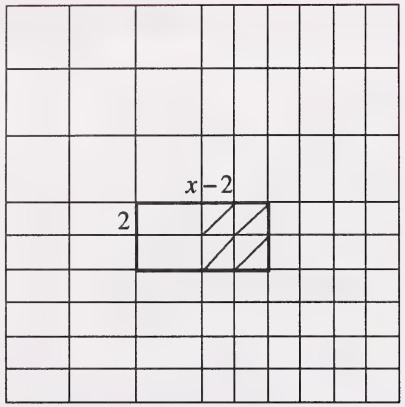


or



$\therefore x^2 - 4x = x(x-4)$

d.



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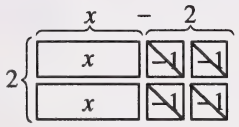


$\therefore 2x-4=2(x-2)$

2. a. $3x+9=3(x+3)$

b. $8y-6=2(4y-3)$

c. $x^2+xy=x(x+y)$

d. $ax^2+ay=a(x^2+y)$

e. $21x^2+14x+7=7(3x^2+2x+1)$

f. $4x^3y+8x^2y^2-4xy^3=4xy(x^2+2xy-y^2)$

g. $6x^4y^4+9x^3y^3-3x^2y^2=3x^2y^2(2x^2y^2+3xy-1)$

h. $25x^3yz^2-10x^2y^2z+15xy^3z^3=5xyz(5x^2z-2xy+3y^2z^2)$

3. $A=l\times w$

$16x^2-32x=8x\times w$

Now factor the left side to get $8x$ as one of the factors.

$16x^2-32x=8x(2x-4)$

The length of the other side is $2x-4$.

Notice that the expression was not factored completely since one of the sides must be $8x$.

Activity 2

Factor a polynomial that has a common binomial factor.

1. $ab^2 - b^2 - 2a + 2 = (ab^2 - b^2) + (-2a + 2)$
$= b^2(a-1) - 2(a-1)$
$= (b^2 - 2)(a-1)$

   -2 must be factored from the second pair in order to obtain a common binomial factor of $(a-1)$.

2. $dx + dy + 2bx + 2by = (dx + dy) + (2bx + 2by)$
$= d(x+y) + 2b(x+y)$
$= (d + 2b)(x+y)$

3. $am^2 + amn - km - kn = (am^2 + amn) + (-km - kn)$
$= am(m+n) - k(m+n)$
$= (am - k)(m+n)$

4. $yb - b - 2y + 2 = (yb - b) + (-2y + 2)$
$= b(y-1) - 2(y-1)$
$= (b-2)(y-1)$

5. $a^3 - y - a^2 + ay = (a^3 - a^2) + (-y + ay)$
$= a^2(a-1) + y(-1+a)$
$= (a^2 + y)(a-1)$

   Note that $(-1+a) = (a-1)$.

6. $6 - 3a - 2b + ab = (6 - 3a) + (-2b + ab)$
$= 3(2-a) - b(2-a)$
$= (3-b)(2-a)$

7. $3m^3 + 2m + 12m^2 + 8 = (3m^3 + 12m^2) + (2m + 8)$
$= 3m^2(m+4) + 2(m+4)$
$= (3m^2 + 2)(m+4)$

8. $2m^3 + 2m - 3m^2 - 3 = (2m^3 + 2m) + (-3m^2 - 3)$
$= 2m(m^2 + 1) - 3(m^2 + 1)$
$= (2m - 3)(m^2 + 1)$

9. $ae + bd - ad - be = (ae - ad) + (bd - be)$
$= a(e-d) - b(e-d)$
$= (a-b)(e-d)$

10. $ab+3a-b-3=(ab-b)+(3a-3)$
$=b(a-1)+3(a-1)$
$=(b+3)(a-1)$

Extra Help

1. $6ce+6de+6cf+6df=6(ce+de+cf+df)$
$=6[e(c+d)+f(c+d)]$
$=6(e+f)(c+d)$

2. $4acx+4adx+4bcx+4bdx=4x(ac+ad+bc+bd)$
$=4x[a(c+d)+b(c+d)]$
$=4x(a+b)(c+d)$

3. $6a^2b+9axy+6abx+9x^2y=3(2a^2b+3axy+2abx+3x^2y)$
$=3(2a^2b+2abx+3axy+3x^2y)$
$=3[2ab(a+x)+3xy(a+x)]$
$=3(2ab+3xy)(a+x)$

Extensions

1. $2a^2-3a+3b-2ab+2ac-3c=2a^2-2ab+2ac-3a+3b-3c$
$=2a(a-b+c)-3(a-b+c)$
$=(2a-3)(a-b+c)$

2. $6xy-y-6xz+z-6x+1=6xy-6xz-6x-y+z+1$
$=6x(y-z-1)-1(y-z-1)$
$=(6x-1)(y-z-1)$

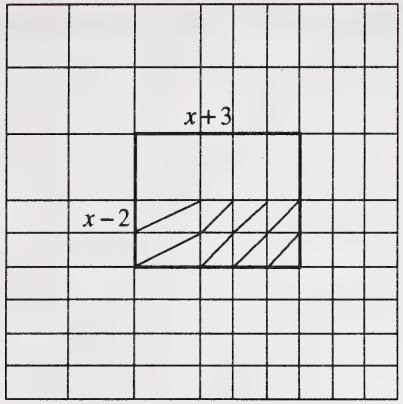
3. $2acx-2ady+adx-4acy+2bcx+bdx-2bdy-4bcy$
$=2acx+2bcx-4acy-4bcy+adx+bdx-2ady-2bdy$
$=(2c)(ax+bx-2ay-2by)+d(ax+bx-2ay-2by)$
$=(2c+d)(ax+bx-2ay-2by)$
$=(2c+d)[x(a+b)-2y(a+b)]$
$=(2c+d)(x-2y)(a+b)$

Exploring Topic 2

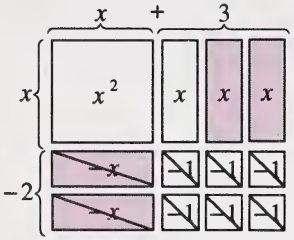
Activity 1

Factor trinomials of the form ax^2+bx+c, where $a, b, c, \in I$.

1. a.

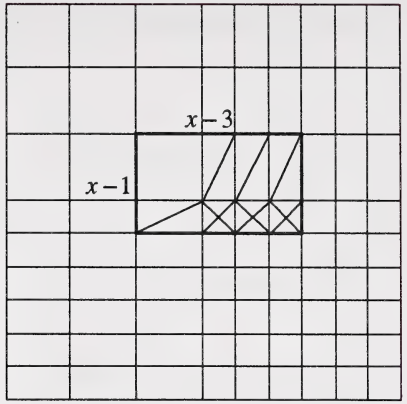


or



$\therefore x^2 + x - 6 = (x-2)(x+3)$

b.



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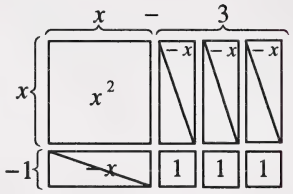
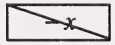
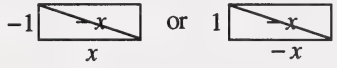
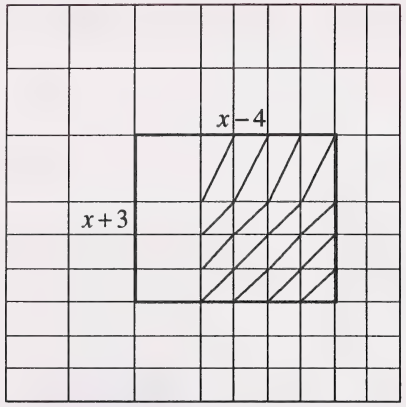


$\therefore x^2 - 4x + 3 = (x-1)(x-3)$

Note that  means that the area of this rectangle is $-x$. Its dimensions may be as follows:



c.



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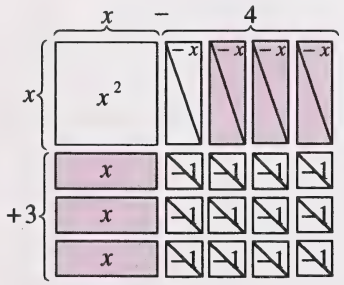
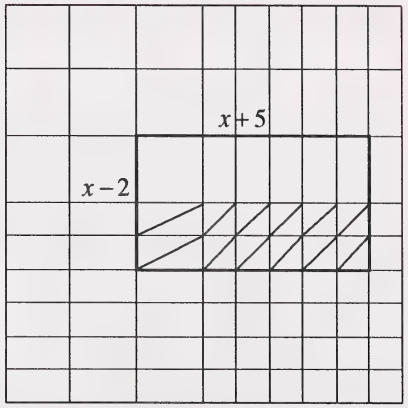


$\therefore x^2 - x - 12 = (x+3)(x-4)$

d.



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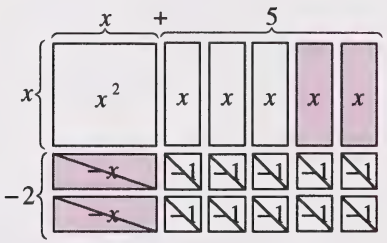
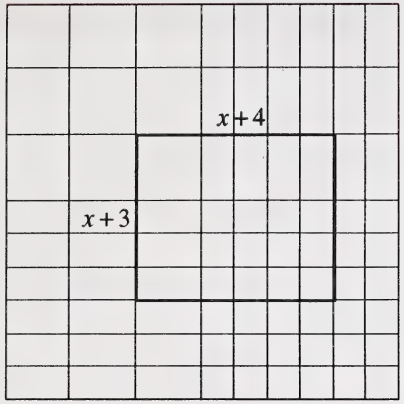


$\therefore x^2 + 3x - 10 = (x-2)(x+5)$

e.



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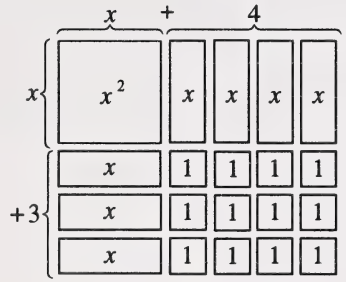
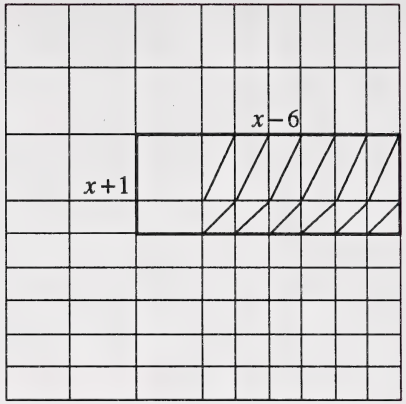


$\therefore x^2 + 7x + 12 = (x+3)(x+4)$

f.



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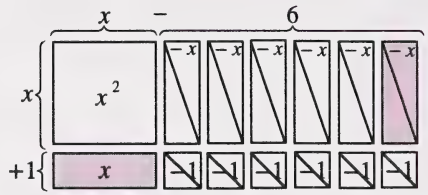


$\therefore x^2 - 5x - 6 = (x+1)(x-6)$

2. a. $x^2 - 4x + 3 = (x-\mathbf{3})(x-\mathbf{1})$

b. $y^2 - 6y - 7 = (y-\mathbf{7})(y+\mathbf{1})$

c. $x^2 - 4x - 32 = (x-\mathbf{8})(x+\mathbf{4})$

d. $y^2+9y+14=(y+7)(y+2)$

e. $x^2-11x+30=(x-6)(x-5)$

f. $y^2-4y-12=(y-6)(y+2)$

g. $x^2+2x-15=(x+5)(x-3)$

h. $y^2-3y-28=(y-7)(y+4)$

i. $x^2-9x+20=(x-5)(x-4)$

j. $x^2+2x-35=(x+7)(x-5)$

k. $y^2-9y-22=(y-11)(y+2)$

l. $x^2-9x+18=(x-6)(x-3)$

3. a. $x^2-6x+8=(x-4)(x-2)$

b. $y^2-2y-3=(y-3)(y+1)$

c. $x^2-10x+21=(x-7)(x-3)$

d. $y^2-3y-54=(y-9)(y+6)$

e. $a^2+8a+7=(a+7)(a+1)$

f. $x^2-13x+40=(x-8)(x-5)$

g. $y^2+18y+72=(y+12)(y+6)$

h. $x^2+11x-42=(x+14)(x-3)$

4. a. $-2x^2+2x+4=-2\left(x^2-x-2\right)$

$=-2(x-2)(x+1)$

b. $-x^2-4x+21=(-1)\left(x^2+4x-21\right)$

$=(-1)(x+7)(x-3)$

It is easier to factor when the coefficient of x^2 is positive.

c. $2y^2+12y+10=2\left(y^2+6y+5\right)$

$=2(y+5)(y+1)$

d. $15x^2-45x+30=15\left(x^2-3x+2\right)$

$=15(x-2)(x-1)$

e. $6xy^2-12xy-18x=6x\left(y^2-2y-3\right)$

$=6x(y-3)(y+1)$

f. $-3x^2-24x-21=(-3)(x^2+8x+7)$

$=(-3)(x+7)(x+1)$

g. $2ax^2+22ax+20a=2a(x^2+11x+10)$

$=2a(x+10)(x+1)$

h. $3x^3-12x^2+12x=3x(x^2-4x+4)$

$=3x(x-2)(x-2)$

$=3x(x-2)^2$

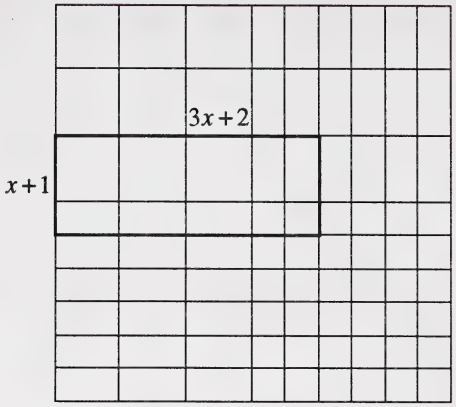
i. $4x^2a^2+8x^2a-12x^2=4x^2(a^2+2a-3)$

$=4x^2(a+3)(a-1)$

j. $bx^2+2bx-35b=b(x^2+2x-35)$

$=b(x+7)(x-5)$

5. a.



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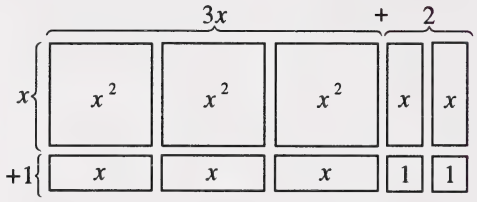
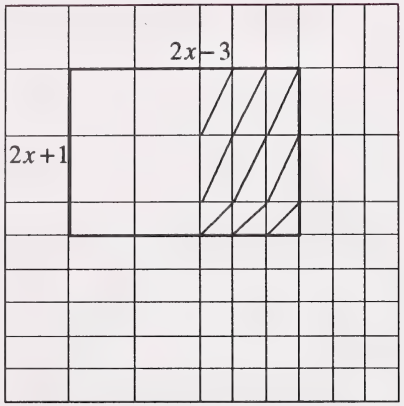


$\therefore 3x^2+5x+2=(x+1)(3x+2)$

b.



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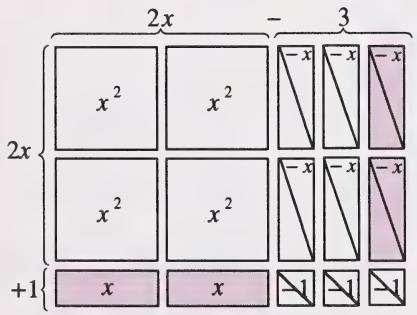
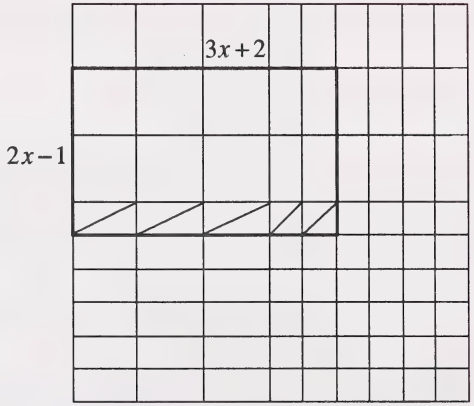


$\therefore 4x^2 - 4x - 3 = (2x+1)(2x-3)$

c.



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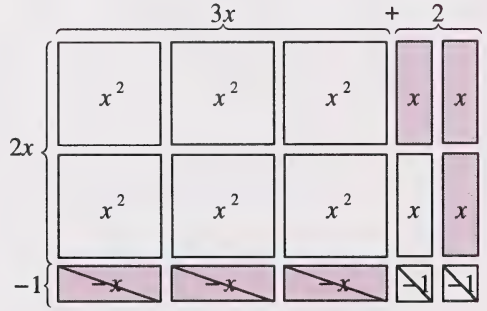
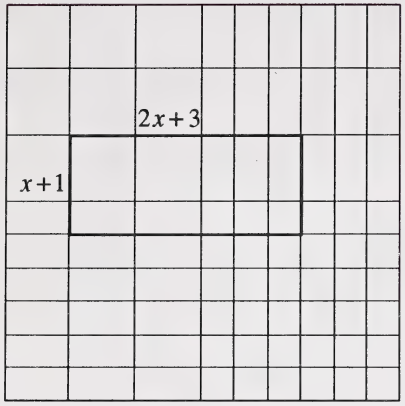


$\therefore 6x^2 + x - 2 = (2x-1)(3x+2)$

d.



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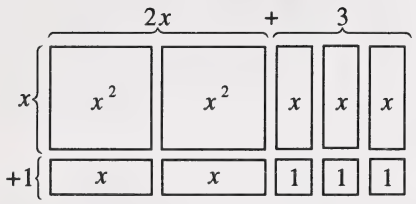
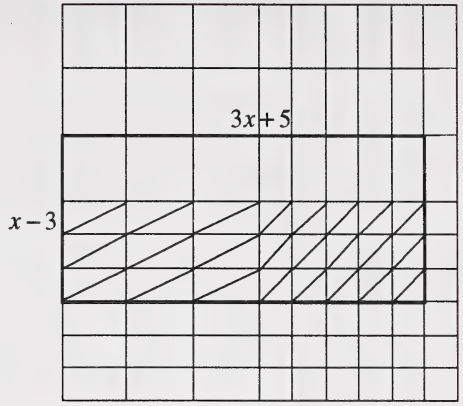


$\therefore 2x^2+5x+3=(x+1)(2x+3)$

e.



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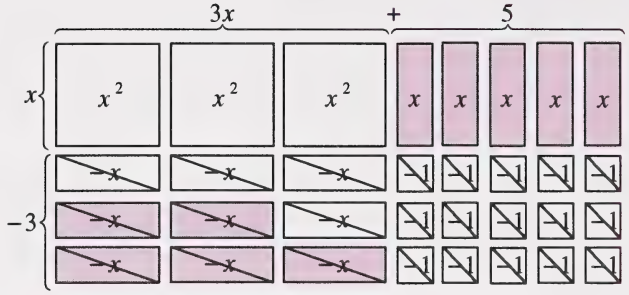
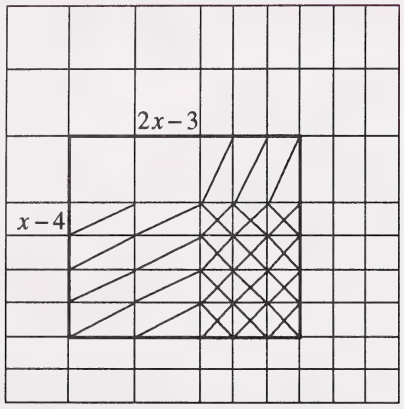


$\therefore 3x^2-4x-15=(x-3)(3x+5)$

f.



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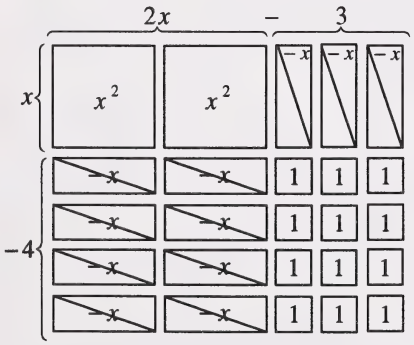


$\therefore 2x^2 - 11x + 12 = (x-4)(2x-3)$

6. a. $2x^2 + 13x + 15 = (2x+3)(x+5)$

b. $12x^2 - 29x + 15 = (4x-3)(3x-5)$

c. $8x^2 + 10x - 3 = (4x-1)(2x+3)$

d. $10x^2 - 29x + 10 = (5x-2)(2x-5)$

e. $15x^2 - 29x - 14 = (5x+2)(3x-7)$

f. $8x^2 + 14x - 15 = (4x-3)(2x+5)$

g. $8x^2 + 8x - 30 = 2\left(4x^2 + 4x - 15\right)$
$= 2(2x+5)(2x-3)$

h. $6x^2 + 7x - 5 = (3x+5)(2x-1)$

i. $9x^2 - 36x + 27 = 9\left(x^2 - 4x + 3\right)$
$= 9(x-3)(x-1)$

j. $2x^2 + x - 15 = (2x-5)(x+3)$

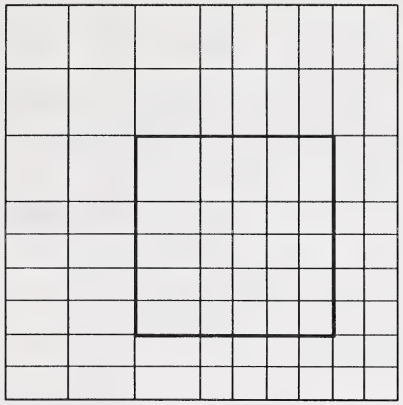
k. $4x^2 - 17x - 15 = (4x+3)(x-5)$

l. $9x^2 + 9x - 10 = (3x+5)(3x-2)$

Activity 2

Recognize the expansion of the square of a binomial as a special case.

1. a.

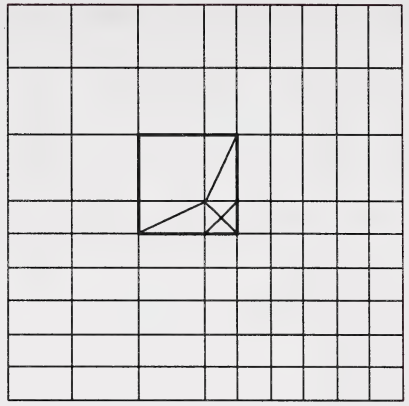


or

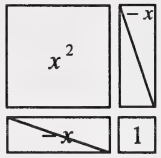
x^2	x	x	x	x
x	1	1	1	1
x	1	1	1	1
x	1	1	1	1
x	1	1	1	1

Since this trinomial forms a square, $x^2+8x+16$ is a perfect square trinomial.

b.

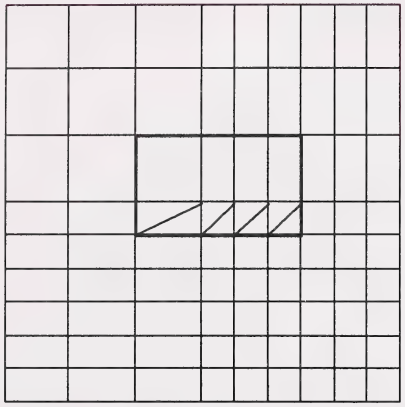


or



Since this trinomial forms a square, x^2-2x+1 is a perfect square trinomial.

c.



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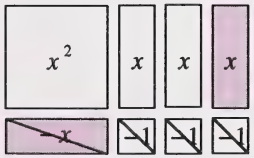
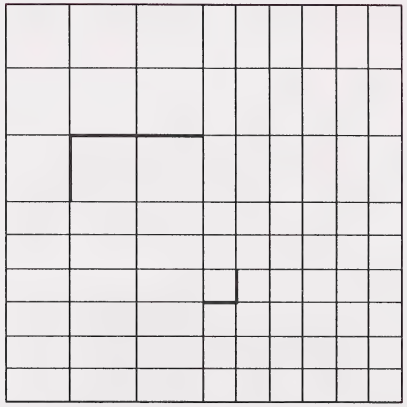


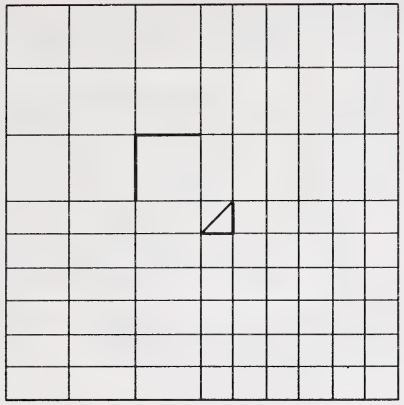
Since this trinomial does not form a square, x^2+2x-3 is not a perfect square trinomial.

d.



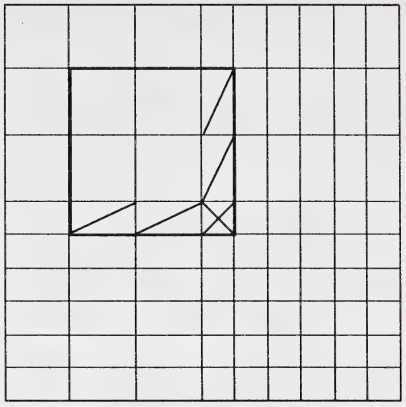
Since this trinomial does not form a square, $2x^2+12x+3$ is not a perfect square trinomial. Since the trinomial cannot be represented on a grid, it cannot be factored.

e.



Since this trinomial does not form a square, $x^2 + 2x - 1$ is not a perfect square trinomial. Since the trinomial cannot be represented on a grid, it cannot be factored.
Note: One of the unit lengths must be negative to produce a product of -1.

f.



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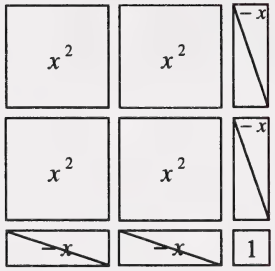
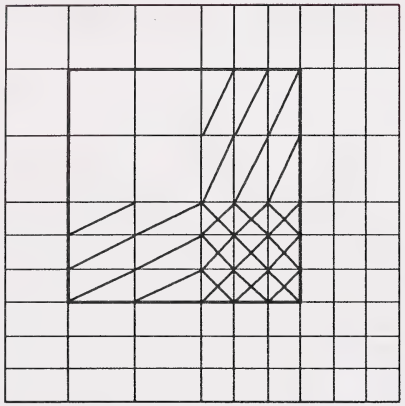


Since this trinomial forms a square, $4x^2 - 4x + 1$ is a perfect square trinomial.

g.



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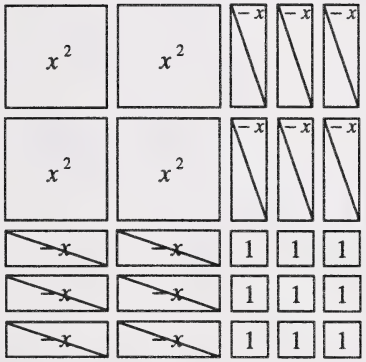
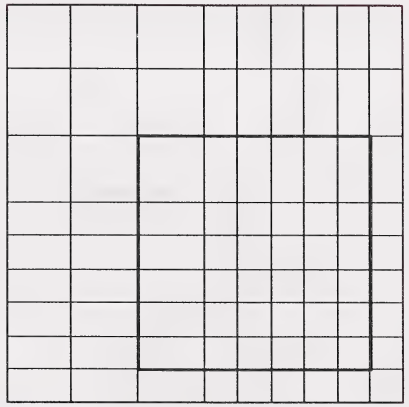


Since this trinomial forms a square, $4x^2 - 12x + 9$ is a perfect square trinomial.

h.



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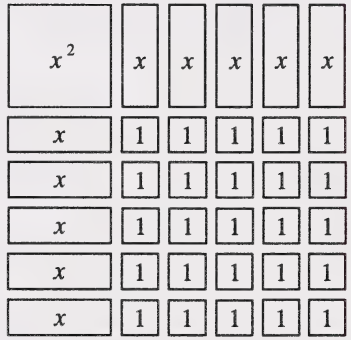


Since this trinomial forms a square, $x^2 + 10x + 25$ is a perfect square trinomial.

2. In order to decide if the trinomials are perfect square trinomials, two conditions must be satisfied.

   - Are the first and last terms perfect squares?
   - Is the middle term equal to twice the product of the square roots of the first and last terms.

   a. $\sqrt{16x^2} = 4x$ and $\sqrt{9} = 3$

   The first and last terms of $16x^2 - 12x + 9$ are perfect squares.

   $2(4x)(3) = 24x$

   The middle term is not equal to twice the product of the square roots of the first and last terms.

   Since both conditions are not satisfied, $16x^2 - 12x + 9$ is not a perfect square trinomial.

   b. $\sqrt{4x^4} = 2x^2$ and $\sqrt{9y^4} = 3y^2$

   The first and last terms of $4x^4 + 12x^2y^2 + 9y^4$ are perfect squares.

   $2\left(2x^2\right)\left(3y^2\right) = 12x^2y^2$

   The middle term is equal to twice the product of the square roots of the first and last terms.

   Since both conditions are satisfied, $4x^4 + 12x^2y^2 + 9y^4$ is a perfect square trinomial.

   c. $\sqrt{x^2} = x$ and $\sqrt{25} = 5$

   The first and last terms of $x^2 + 10x + 25$ are perfect squares.

   $2(x)(5) = 10x$

   The middle term is equal to twice the product of the square roots of the first and last terms.

   Since both conditions are satisfied, $x^2 + 10x + 25$ is a perfect square trinomial.

d. $\sqrt{25x^2} = 5x$ and $\sqrt{16} = 4$

The first and last terms of $25x^2 - 40x + 16$ are perfect squares.

$2(5x)(4) = 40x$

The middle term is equal to twice the product of the square roots of the first and last terms.

Therefore, $25x^2 - 40x + 16$ is a perfect square trinomial.

e. $\sqrt{x^2} = x$ and $\sqrt{16} = 4$

The first and last terms of $x^2 + 4x + 16$ are perfect squares.

$2(x)(4) = 8x$

The middle term is not equal to twice the product of the squares roots of the first and last terms

Therefore, $x^2 + 4x + 16$ is not a perfect square trinomial.

f. $\sqrt{16x^2} = 4x$ and $\sqrt{1} = 1$

The first and last terms of $16x^2 - 8x + 1$ are perfect squares.

$2(4x)(1) = 8x$

The middle term is euqal to twice the product of the square roots of the first and last terms.

Therefore, $16x^2 - 8x + 1$ is a perfect square trinomial.

g. $\sqrt{4x^2} = 2x$ and $\sqrt{4} = 2$

The first and last terms of $4x^2 - 4x + 4$ are perfect squares.

$2(2x)(2) = 8x$

The middle term is not equal to twice the product of the square roots of the first and last terms.

Therefore, $4x^2 - 4x + 4$ is not a perfect square trinomial.

h. $\sqrt{a^2} = a$ and $\sqrt{b^2} = b$

The first and last terms of $a^2 + 2ab + b^2$ are perfect squares.

$2(a)(b) = 2ab$

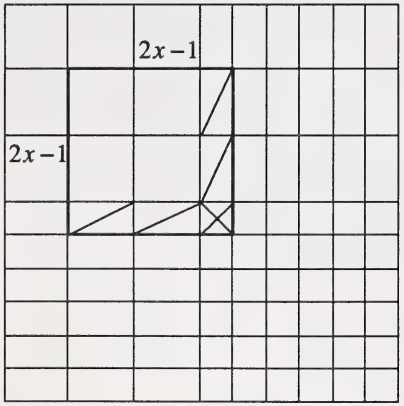
The middle term is equal to twice the product of the square roots of the first and last terms.

Therefore, $a^2 + 2ab + b^2$ is a perfect square trinomial.

Activity 3

Factor a perfect square trinomial.

1. a.



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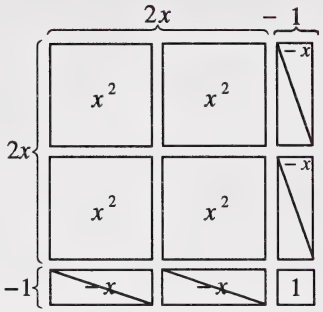
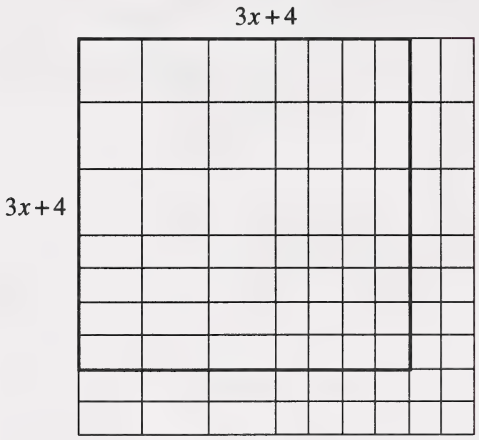


Therefore, $4x^2 - 4x + 1 = (2x - 1)(2x - 1) = (2x - 1)^2$.

b.



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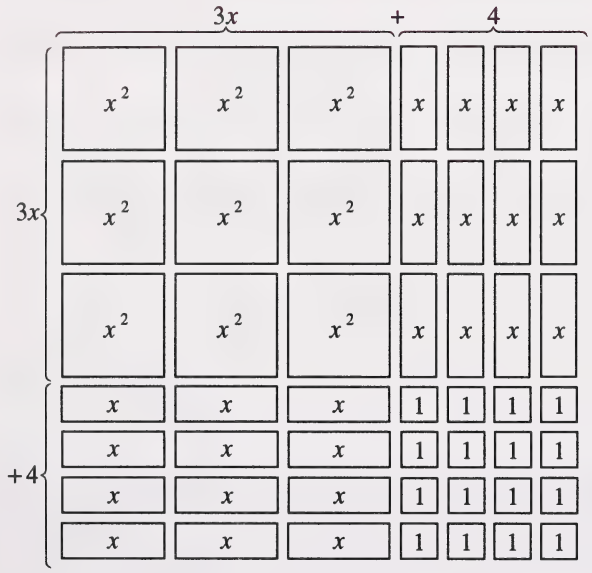
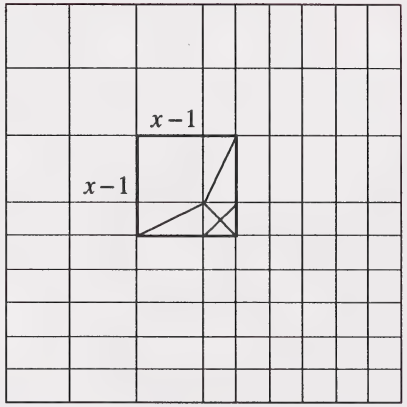


Therefore, $9x^2+24x+16=(3x+4)(3x+4)=(3x+4)^2$.

c.



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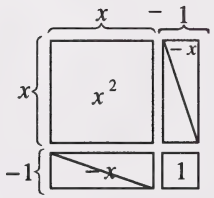
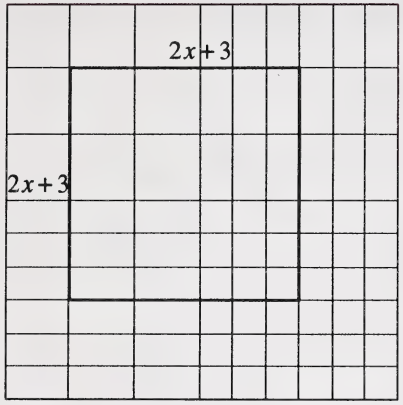


Therefore, $x^2-2x+1=(x-1)(x-1)=(x-1)^2$.

d.



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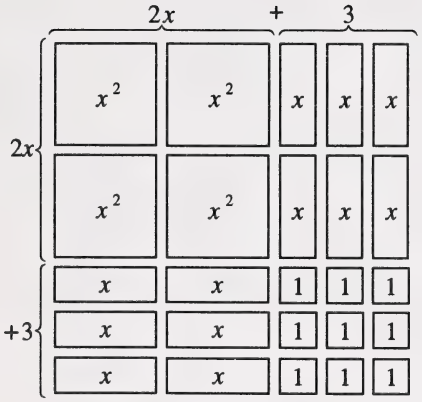
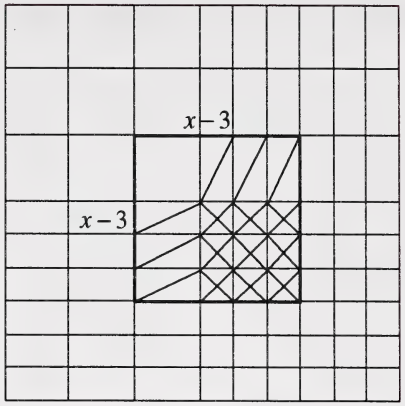


Therefore, $4x^2+12+9=(2x+3)(2x+3)=(2x+3)^2$.

e.



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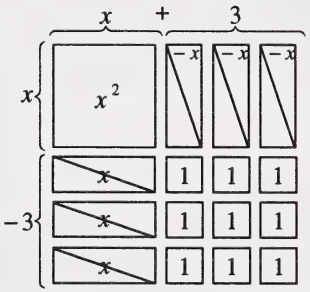
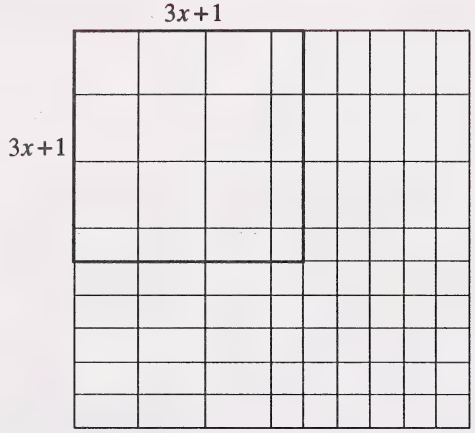


Therefore, $x^2-6x+9=(x-3)(x-3)=(x-3)^2$.

f.



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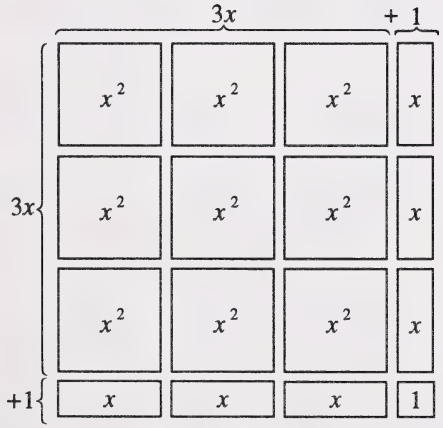
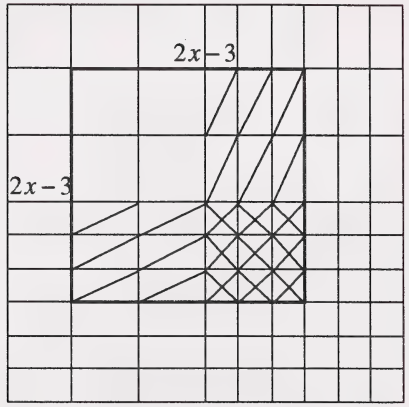


Therefore, $9x^2+6x+1=(3x+1)(3x+1)=(3x+1)^2$.

g.



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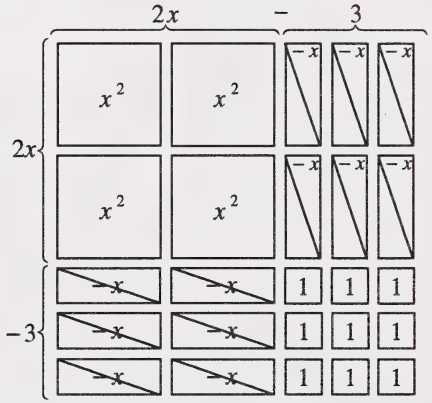
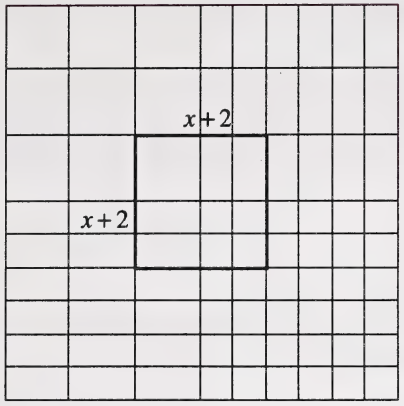


Therefore, $4x^2-12x+9=(2x-3)(2x-3)=(2x-3)^2$.

h.



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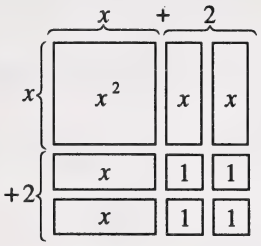


Therefore, $x^2+4x+4=(x+2)(x+2)=(x+2)^2$.

2. a. $x^2+14x+49=(x+7)(x+7)$

$=(x+7)^2$

b. $16x^2-8x+1=(4x-1)^2$

c. $144x^2+120x+25=(12x+5)^2$

d. $243m^2n^2+54mn+3=3\left(81m^2n^2+18mn+1\right)$

$=3(9mn+1)(9mn+1)$

$=3(9mn+1)^2$

e. $16x^2-24xy+9y^2=(4x-3y)^2$

f. $x^4+2x^2y^2+y^4=\left(x^2+y^2\right)^2$

g. $x^2+x+\frac{1}{4}=\left(x+\frac{1}{2}\right)^2$

h. $x^2+18x+81=(x+9)^2$

Activity 4

Recognize the product of conjugate binomials as a special case.

In order to decide if a binomial is a difference of squares, two conditions must be satisfied.

- Are there only two terms, one of which, is negative?
- Are both terms perfect squares?

1. There are two terms in $36a^2 - 25b^4$. The second term is negative. Both terms are perfect squares.

   $\sqrt{36a^2} = 6a$ and $\sqrt{25b^4} = 5b^2$

   Since both conditions are satisfied, $36a^2 - 25b^4$ is a difference of squares.

2. There are two terms in $y^2 - 16$. The second term is negative. Both terms are perfect squares.

   $\sqrt{y^2} = y$ and $\sqrt{16} = 4$

   Therefore, $y^2 - 16$ is a difference of squares.

3. There are two terms in $36x^2 + 9$. Both terms are positive.

   Therefore, $36x^2 + 9$ is not a difference of squares.

4. There are two terms in $4a^2b^2 - \frac{1}{9}$. The second term is negative. Both terms are perfect squares.

   $\sqrt{4a^2b^2} = 2ab$ and $\sqrt{\frac{1}{9}} = \frac{1}{3}$

   Therefore, $4a^2b^2 - \frac{1}{9}$ is a difference of squares.

5. There are two terms in $100x^2 - 400y^2$. The second term is negative. Both terms are perfect squares.

   $\sqrt{100x^2} = 10x$ and $\sqrt{400y^2} = 20y$

   Therefore, $100x^2 - 400y^2$ is a difference of squares.

6. There are two terms in $(x+2)^2 - (x+3)^2$. The second term is negative. Both terms are perfect squares.

   $\sqrt{(x+2)^2} = (x+2)$ and $\sqrt{(x+3)^2} = (x+3)$

   Therefore, $(x+2)^2 - (x+3)^2$ is a difference of squares.

7. There are three terms in $x^2 + 4x + 4$.

   Therefore, $x^2 + 4x + 4$ is not a difference of squares.

8. There are two terms in $5x^3 - 4$. The second term is negative. Both terms are not perfect squares.

   $5x^3$ is not a perfect square.

   Therefore, $5x^3 - 4$ is not a difference of squares.

Activity 5

Factor a difference of squares binomial.

1. $m^6 - n^2 = (m^3 + n)(m^3 - n)$

2. $x^2 - 16 = (x+4)(x-4)$

3. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x+y)(x-y)$

   Note: $x^2 - y^2$ is also a difference of squares and must be factored to $(x+y)(x-y)$.

4. $16s^2 - t^2 = (4s+t)(4s-t)$

5. $y^4 - 1 = (y^2 + 1)(y^2 - 1)$
$= (y^2 + 1)(y+1)(y-1)$

6. $(x^2 + 1)^2 - x^2 = [(x^2 + 1) + x][(x^2 + 1) - x]$
$= (x^2 + x + 1)(x^2 - x + 1)$

   You could substitute a single variable for the complex expression $(x^2 + 1)$. Let $y = (x^2 + 1)$.

   $(x^2 + 1)^2 - x^2 = y^2 - x^2$
   $= (y+x)(y-x)$

   Now replace y with $(x^2 + 1)$.

   $(x^2 + 1 + x)(x^2 + 1 - x) = (x^2 + x + 1)(x^2 - x + 1)$

7. $9y^2 - 49b^2 = (3y + 7b)(3y - 7b)$

8. $\frac{x^2}{36} - 1 = \left(\frac{x}{6} + 1\right)\left(\frac{x}{6} - 1\right)$

Activity 6

Factor polynomials that require a combination of methods in order to be completely factored.

1. a. $6x^2y^2 - 9xy^2 + 3xy = 3xy(2xy - 3y + 1)$

   b. $x^5 - 81x = x(x^4 - 81)$
   $= x(x^2 + 9)(x^2 - 9)$
   $= x(x^2 + 9)(x-3)(x+3)$

c. $(x+y)^2-1=[(x+y)+1][(x+y)-1]$
$=(x+y+1)(x+y-1)$

d. $-5y^2-6y+11=-1(5y^2+6y-11)$
$=-1(5y+11)(y-1)$

e. $5-80a^2=5(1-16a^2)$
$=5(1+4a)(1-4a)$

f. $16x^2-15x-1=(16x+1)(x-1)$

g. $xy-x^3y^3=xy(1-x^2y^2)$
$=xy(1+xy)(1-xy)$

h. $(a^2-ab)^2-16b^4=[(a^2-ab)+4b^2][(a^2-ab)-4b^2]$
$=(a^2-ab+4b^2)(a^2-ab-4b^2)$

i. $12y^2+5y-3=(4y+3)(3y-1)$

j. $64y^2-49x^2=(8y-7x)(8y+7x)$

k. $15a^3b-11ab^3=ab(15a^2-11b^2)$
$=ab(\sqrt{15}a+\sqrt{11}b)(\sqrt{15}a-\sqrt{11}b)$

l. $30x^2+10x-100=10(3x^2+x-10)$
$=10(3x-5)(x+2)$

m. $x^3+5x^2-36x=x(x^2+5x-36)$
$=x(x+9)(x-4)$

n. $9y^4+6y^2+1=(3y^2+1)(3y^2+1)$
$=(3y^2+1)^2$

2. a. $SA=\pi rs+\pi r^2$ (Factor the common factor πr.)
$=\pi r(s+r)$

b. Substitute $\pi=3.14$, $r=3$ cm, and $s=5$ cm in the factored formula.

$SA=\pi r(s+r)$
$=(3.14)(3)(5+3)$
$=(3.14)(3)(8)$
$=75.36\text{ cm}^2$

The surface area of the cone is 75.36 cm^2.

Enter	Display
5	5
+	5
3	3
=	8
×	8
3.14	3.14
×	25.12
3	3
=	75.36

Enter	Display
3.14	3.14
×	3.14
(	0
12	12
−	12
7	7
)	5
×	15.7
(	0
12	12
+	12
7	7
)	19
=	298.3

3. a. $A = \pi R^2 - \pi r^2$

$= \pi\left(R^2 - r^2\right)$ $\left(R^2 - r^2\right)$ is a difference of squares.

$= \pi(R-r)(R+r)$

b. $A = \pi(R-r)(R+r)$

$= 3.14(12-7)(12+7)$

$= 3.14(5)(19)$

$= 298.3 \text{ cm}^2$

The area of the shaded region is 298.3 cm^2.

Extra Help

1. $x^2+3x-10$

$$-10x^2 \left|\begin{array}{r} 5x \\ -2x \\ \hline 3x \end{array}\right.$$

$$x^2+5x-2x-10 = x(x+5)-2(x+5)$$
$$= (x-2)(x+5)$$

2. $x^2-8x+12$

$$12x^2 \left|\begin{array}{r} -6x \\ -2x \\ \hline -8x \end{array}\right.$$

$$x^2-6x-2x+12 = x(x-6)-2(x-6)$$
$$= (x-2)(x-6)$$

3. $x^2+14x+49$

$$49x^2 \left|\begin{array}{r} 7x \\ 7x \\ \hline 14x \end{array}\right.$$

$$x^2+7x+7x+49 = x(x+7)+7(x+7)$$
$$= (x+7)(x+7)$$
$$= (x+7)^2$$

4. $4x^2+20x+25$

$$100x^2 \left|\begin{array}{r} 10x \\ 10x \\ \hline 20x \end{array}\right.$$

$$4x^2+10x+10x+25 = 2x(2x+5)+5(2x+5)$$
$$= (2x+5)(2x+5)$$
$$= (2x+5)^2$$

5. $4x^2-12x+9$

$$36x^2 \left|\begin{array}{r} -6x \\ -6x \\ \hline -12x \end{array}\right.$$

$$4x^2-6x-6x+9 = 2x(2x-3)-3(2x-3)$$
$$= (2x-3)(2x-3)$$
$$= (2x-3)^2$$

6. x^2-6x+8

$$8x^2 \left|\begin{array}{r} -4x \\ -2x \\ \hline -6x \end{array}\right.$$

$$x^2-4x-2x+8 = x(x-4)-2(x-4)$$
$$= (x-2)(x-4)$$

7. x^2+4x-5

$-5x^2$ | $5x$, $-x$ | $4x$

$x^2+5x-x-5=x(x+5)-1(x+5)$

$=(x-1)(x+5)$

8. $3x^2+16x-35$

$-105x^2$ | $21x$, $-5x$ | $16x$

$3x^2+21x-5x-35=3x(x+7)-5(x+7)$

$=(3x-5)(x+7)$

Extensions

1. a. $\dfrac{x^2-2x+1}{x-1}=\dfrac{\overset{1}{\cancel{(x-1)}}(x-1)}{\underset{1}{\cancel{(x-1)}}}$

$=x-1$

b. $\dfrac{x^2-4}{x^2+4x+4}=\dfrac{\overset{1}{\cancel{(x+2)}}(x-2)}{\underset{1}{\cancel{(x+2)}}(x+2)}$

$=\dfrac{x-2}{x+2}$

c. $\dfrac{x^2-5x+6}{x^2+2x-15}=\dfrac{\overset{1}{\cancel{(x-3)}}(x-2)}{(x+5)\underset{1}{\cancel{(x-3)}}}$

$=\dfrac{x-2}{x+5}$

d. $\dfrac{y^2-1}{y-1}=\dfrac{\overset{1}{\cancel{(y-1)}}(y+1)}{\underset{1}{\cancel{(y-1)}}}$

$=y+1$

2. a. $\dfrac{4}{xy}+\dfrac{5}{x}=\dfrac{4}{xy}+\dfrac{5y}{xy}$

$=\dfrac{4+5y}{xy}$

b. $\dfrac{2}{x^2-1}+\dfrac{3}{x^2-2x+1}$

$=\dfrac{2}{(x-1)(x+1)}+\dfrac{3}{(x-1)(x-1)}$

$=\dfrac{2(x-1)}{(x-1)(x+1)(x-1)}+\dfrac{3(x+1)}{(x-1)(x-1)(x+1)}$

$=\dfrac{2(x-1)+3(x+1)}{(x-1)(x-1)(x+1)}$

$=\dfrac{2x-2+3x+3}{(x-1)(x-1)(x+1)}$

$=\dfrac{5x+1}{(x-1)(x-1)(x+1)}$

c. $\dfrac{x^2-9}{x^2-5x+6}-\dfrac{3}{x-2}=\dfrac{\cancel{(x-3)}^{1}(x+3)}{\cancel{(x-3)}_{1}(x-2)}-\dfrac{3}{x-2}$

$=\dfrac{x+3-3}{x-2}$

$=\dfrac{x}{x-2}$

d. $\dfrac{2x}{4x^2+8x}-\dfrac{x}{2x+4}=\dfrac{\cancel{2x}^{1}}{\cancel{2x}_{1}(2x+4)}-\dfrac{x}{2x+4}$

$=\dfrac{1}{2x+4}-\dfrac{x}{2x+4}$

$=\dfrac{1-x}{2x+4}$

3. a. Let $99=100-1$ and $101=100+1$.

$\therefore 99\times101=(100-1)(100+1)$

$=100^2-1^2$

$=10\,000-1$

$=9999$

b. Let $499=500-1$ and $501=500+1$.

$\therefore 499\times501=(500-1)(500+1)$

$=500^2-1^2$

$=250\,000-1$

$=249\,999$

Exploring Topic 3

Activity 1

Find the rational zeros of an integral quadratic polynomial in one variable by factoring.

1. $x^2+2x-3=0$

$(x+3)(x-1)=0$

$x+3=0$ or $x-1=0$

$x=-3$ $\qquad$ $x=1$

Therefore, the zeros are $x=-3$ and $x=1$.

2. $25k^2-49=0$

$(5k+7)(5k-7)=0$

$5k+7=0$ or $5k-7=0$

$5k=-7$ $\qquad$ $5k=7$

$k=-\dfrac{7}{5}$ $\qquad$ $k=\dfrac{7}{5}$

Therefore, the zeros are $k=-\frac{7}{5}$ and $k=\frac{7}{5}$.

3. $2y^3 - 50y = 0$

$2y(y^2 - 25) = 0$

$2y(y+5)(y-5) = 0$

$2y = 0$ or $y+5 = 0$ or $y-5 = 0$

$y = 0$ $\quad y = -5$ $\quad y = 5$

Therefore, the zeros are $y = 0$, $y = -5$, and $y = 5$.

4. $3x^2 - x - 2 = 0$

$(3x+2)(x-1) = 0$

$3x+2 = 0$ or $x-1 = 0$

$3x = -2$ $\quad x = 1$

$x = -\frac{2}{3}$

Therefore, the zeros are $x = -\frac{2}{3}$ and $x = 1$.

5. $2k^2 - 5k - 3 = 0$

$(2k+1)(k-3) = 0$

$2k+1 = 0$ or $k-3 = 0$

$2k = -1$ $\quad k = 3$

$k = -\frac{1}{2}$

Therefore, the zeros are $k = -\frac{1}{2}$ and $k = 3$.

6. $x^2 + 7x - 18 = 0$

$(x+9)(x-2) = 0$

$x+9 = 0$ or $x-2 = 0$

$x = -9$ $\quad x = 2$

Therefore, the zeros are $x = -9$ and $x = 2$.

7. $3k^2 + k - 30 = 0$

$(3k+10)(k-3) = 0$

$3k+10 = 0$ or $k-3 = 0$

$3k = -10$ $\quad k = 3$

$k = -\frac{10}{3}$

Therefore, the zeros are $k = -\frac{10}{3}$ and $k = 3$.

8. $6y^2 - 13y + 6 = 0$

$(3y-2)(2y-3)=0$

$3y-2=0$ or $2y-3=0$

$3y=2$ $\quad$ $2y=3$

$y=\frac{2}{3}$ $\quad$ $y=\frac{3}{2}$

Therefore, the zeros are $y=\frac{2}{3}$ and $y=\frac{3}{2}$.

9. $2y^2 + 5y + 2 = 0$

$(2y+1)(y+2)=0$

$2y+1=0$ or $y+2=0$

$2y=-1$ $\quad$ $y=-2$

$y=-\frac{1}{2}$

Therefore, the zeros are $y=-\frac{1}{2}$ and $y=-2$.

10. $x^2 - 2x + 1 = 0$

$(x-1)(x-1)=0$

$x-1=0$ or $x-1=0$

$x=1$ $\quad$ $x=1$

Note: When the solutions are the same, you only need to write it once in the final statement.

The zero is $x=1$.

Activity 2

Solve quadratic equations with integral coefficients in one variable by factoring.

1. $3x^3 + 6x^2 = 9x$

$3x^3 + 6x^2 - 9x = 0$

$3x(x^2 + 2x - 3) = 0$

$3x(x+3)(x-1)=0$

$3x=0$ or $x+3=0$ or $x-1=0$

$x=0$ $\quad$ $x=-3$ $\quad$ $x=1$

The roots are $x=0$, $x=1$, and $x=-3$.

2. $x^2 = 4x + 45$

$x^2 - 4x - 45 = 0$

$(x+5)(x-9)=0$

$x+5=0$ or $x-9=0$

$x=-5$ $\quad$ $x=9$

The roots are $x=-5$ and $x=9$.

3. $18 = x^2 + 3x$

$-x^2 - 3x + 18 = 0$ (Multiply both sides by -1.)

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x+6=0$ or $x-3=0$

$x=-6$ $x=3$

The roots are $x=3$ and $x=-6$.

4. $x^2 + 14 = -9x$

$x^2 + 9x + 14 = 0$

$(x+7)(x+2) = 0$

$x+7=0$ or $x+2=0$

$x=-7$ $x=-2$

The roots are $x=-7$ and $x=-2$.

5. $x^4 + 7x^3 + 12x^2 = 0$

$x^2(x^2 + 7x + 12) = 0$

$x^2(x+4)(x+3) = 0$

$x^2 = 0$ or $x+4=0$ or $x+3=0$

$x=0$ $x=-4$ $x=-3$

The roots are $x=0$, $x=-3$, and $x=-4$.

6. $x^2 + 6x = -9$

$x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x+3=0$ or $x+3=0$

$x=-3$ $x=-3$

The root is $x=-3$.

7. $x^2 + 25 = -10x$

$x^2 + 10x + 25 = 0$

$(x+5)(x+5) = 0$

$x+5=0$ or $x+5=0$

$x=-5$ $x=-5$

The root is $x=-5$.

8. $2x^2 = x+3$

$2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

$2x-3=0$ or $x+1=0$

$2x=3$ $\quad x=-1$

$x=\frac{3}{2}$

The roots are $x=\frac{3}{2}$ and $x=-1$.

9. $2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$

$(2x-1)(x+4) = 0$

$2x-1=0$ or $x+4=0$

$2x=1$ $\quad x=-4$

$x=\frac{1}{2}$

The roots are $x=\frac{1}{2}$ and $x=-4$.

10. $x^3 = 49x$

$x^3 - 49x = 0$

$x(x^2 - 49) = 0$

$x(x+7)(x-7) = 0$

$x=0$ or $x+7=0$ or $x-7=0$

$x=-7$ $\quad x=7$

The roots are $x=0$, $x=-7$, and $x=7$.

Activity 3

Solve problems that can be represented by factorable quadratic equations.

1. Let x equal the smaller number.
   Then $x+7$ equals the larger number.

$x^2 + (x+7)^2 = 85$

$x^2 + x^2 + 14x + 49 = 85$

$2x^2 + 14x - 36 = 0$

$2(x^2 + 7x - 18) = 0$

$2(x+9)(x-2) = 0$

$x+9=0 \quad \text{or} \quad x-2=0$

$x=-9 \qquad x=2$

If the smaller number is -9, then the larger number is $-9+7=-2$.

If the smaller number is 2, then the larger number is $2+7=9$.

The numbers are -9 and -2 or 2 and 9.

2. Let x equal one number.
Let y equal the other number.

$x+y=20$

$y=20-x$

$$x^2+(20-x)^2=298$$

$$x^2+400-40x+x^2=298$$

$$2x^2-40x+400=298$$

$$2x^2-40x+102=0$$

$$2\left(x^2-20x+51\right)=0$$

$$2(x-3)(x-17)=0$$

$x-3=0 \quad \text{or} \quad x-17=0$

$x=3 \qquad x=17$

The numbers are 3 and 17.

3. $$h=v_i t+\frac{1}{2}at^2$$

$$12\,000=1540t+\frac{1}{2}(-10)t^2$$

$$12\,000=1540t-5t^2$$

$$5t^2-1540+12\,000=0$$

$$5\left(t^2-308t+2400\right)=0$$

$$5(t-8)(t-300)=0$$

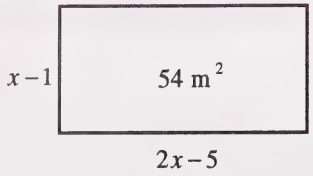
$t-8=0 \quad \text{or} \quad t-300=0$

$t=8 \text{ s} \qquad t=300 \text{ s}$

The times needed to reach this height are 8 s and 300 s.

You can see that your algebra study does have practical applications.

4.



$$\text{Area} = \text{length} \times \text{width}$$
$$54 = (2x-5)(x-1)$$
$$54 = 2x^2 - 2x - 5x + 5$$
$$54 = 2x^2 - 7x + 5$$
$$2x^2 - 7x - 49 = 0$$
$$(2x+7)(x-7) = 0$$

$\therefore 2x+7=0$ or $x-7=0$

$2x = -7$ $\qquad$ $x = 7$

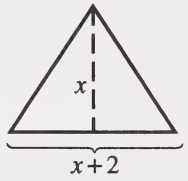
$x = \dfrac{-7}{2}$

When $x = -\frac{7}{2}$, the width of the rectangle, defined by $x-1$, is $-\frac{7}{2} - 1 = -\frac{9}{2}$. Since the width cannot be a negative quantity, the value $x = -\frac{7}{2}$ can be disregarded.

When $x = 7$, the width is $7 - 1 = 6$, and the length, defined by $2x - 5$, is $2(7) - 5 = 14 - 5 = 9$.

Therefore, the sides are 6 m and 9 m.

5. Let x equal the height.
Then, $x+2$ is equal to the base.



$$A = \frac{1}{2}bh$$
$$24 = \frac{1}{2}(x+2)(x)$$
$$24 = \frac{1}{2}\left(x^2 + 2x\right) \quad \text{(Multiply both sides by 2.)}$$
$$48 = x^2 + 2x$$
$$-x^2 - 2x + 48 = 0 \quad \text{(Multiply both sides by } -1\text{.)}$$
$$x^2 + 2x - 48 = 0$$
$$(x+8)(x-6) = 0$$

$x+8=0$ or $x-6=0$

$x = -8$ $\qquad$ $x = 6$

Disregard $x = -8$ because the altitude of a triangle cannot be a negative quantity.

The height must be 6 cm and the base must be $6 + 2 = 8$ cm.

6. Let x equal the number.

$$(3x-2)^2 = 100$$
$$9x^2 - 12x + 4 = 100$$
$$9x^2 - 12x - 96 = 0$$
$$3(3x^2 - 4x - 32) = 0$$
$$3(3x+8)(x-4) = 0$$

$3x+8=0$ or $x-4=0$

$3x=-8$ $\quad$ $x=4$

$x=\frac{-8}{3}$

The two numbers are $\frac{-8}{3}$ and 4.

Extra Help

1. $x-y=5$

$y=x-5$

$$x^2 - y^2 = 45$$
$$x^2 - (x-5)^2 = 45$$
$$x^2 - (x^2 - 10x + 25) = 45$$
$$x^2 - x^2 + 10x - 25 = 45$$
$$10x = 70$$
$$x = 7$$

When $x=7$, $y=7-5$

$=2$

The numbers are 7 and 2.

2. $x-y=5$

$y=x-5$

$$x^2 + y^2 = 97$$
$$x^2 + (x-5)^2 = 97$$
$$x^2 + x^2 - 10x + 25 = 97$$
$$2x^2 - 10x - 72 = 0$$
$$2(x^2 - 5x - 36) = 0$$
$$2(x-9)(x+4) = 0$$

$x-9=0$ or $x+4=0$

$x=9$ $\quad$ $x=-4$

When $x=9$, $y=9-5$

$=4$

When $x=-4$, $y=-4-5$

$=-9$

The two numbers are 9 and 4 or -4 and -9.

3. $x+y=10$

$y=10-x$

$$x^2+y^2=68$$
$$x^2+(10-x^2)=68$$
$$x^2+100-20x+x^2=68$$
$$2x^2-20x+32=0$$
$$2(x^2-10x+16)=0$$
$$2(x-8)(x-2)=0$$

$x=2$ or $x=8$

When $x=2$, $y=8$ and when $x=8$, $y=2$. Therefore, the numbers are 8 and 2.

4. $x+y=15$

$y=15-x$

$$x^2-y^2=45$$
$$x^2-(15-x)^2=45$$
$$x^2-(225-30x+x^2)=45$$
$$x^2-225+30x-x^2=45$$
$$30x-225=45$$
$$30x=270$$
$$x=9$$

When $x=9$, $y=6$. Therefore, the numbers are 9 and 6.

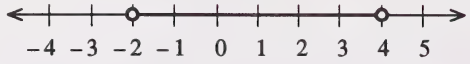
Extensions

1. $x^2-2x-8<0$

$(x-4)(x+2)<0$

$x>4$ False
$x<4$ True
$x>-2$ True
$x<-2$ False

$x>-2$ or $x<4$, $x\in R$

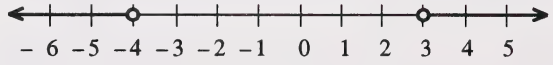


2. $x^2+x-12>0$

$(x+4)(x-3)>0$

$x>-4$ False
$x<-4$ True
$x>3$ True
$x<3$ False

$x<-4$ or $x>3$, $x\in R$

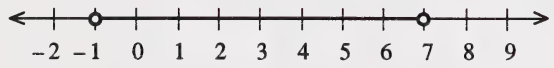


3. $x^2 - 6x - 7 < 0$

$(x-7)(x+1) < 0$

$x > 7$ False
$x < 7$ True
$x > -1$ True
$x < -1$ False

$x > -1$ or $x < 7$, $x \in R$

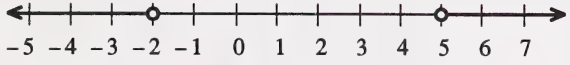


4. $x^2 - 3x - 10 > 0$

$(x-5)(x+2) > 0$

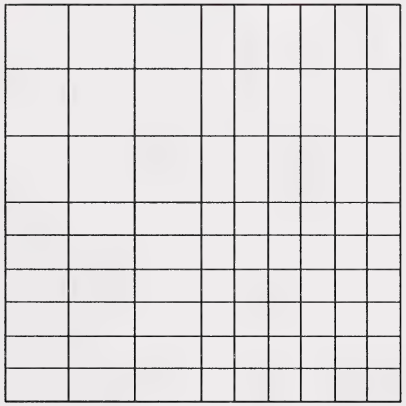
$x > 5$ True
$x < 5$ False
$x > -2$ False
$x < -2$ True

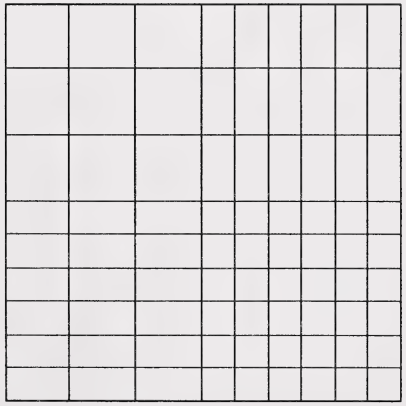
$x < -2$ or $x > 5$, $x \in R$

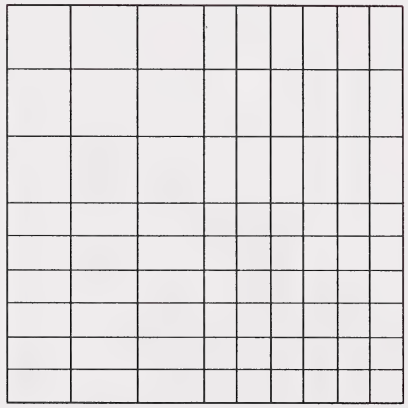


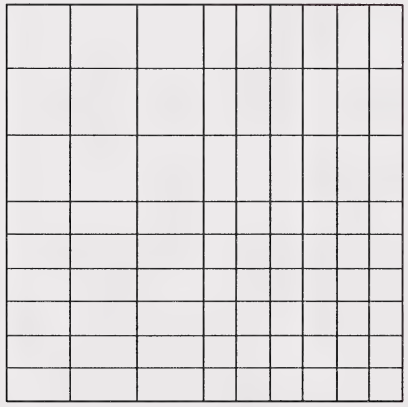
Appendix B
Grids and Tiles

Binomial Grids

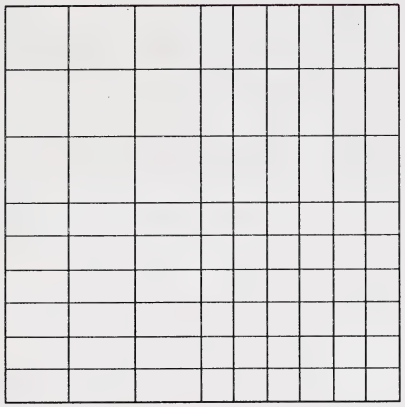


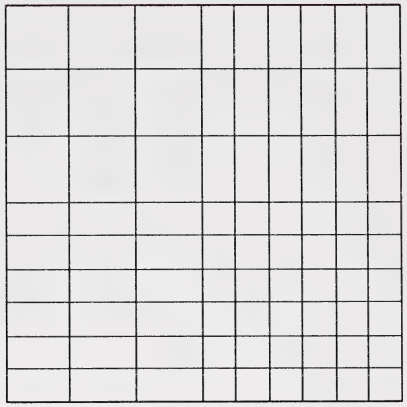


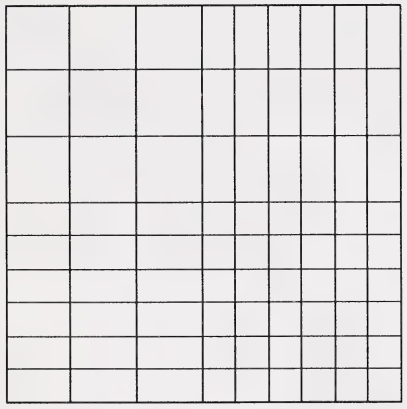


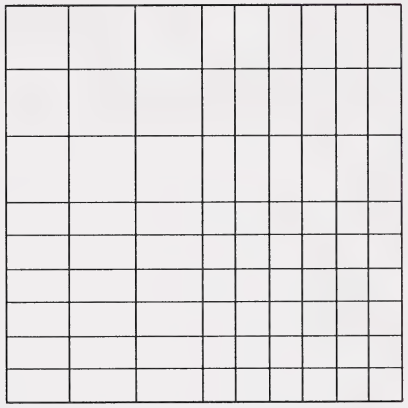


Binomial Grids

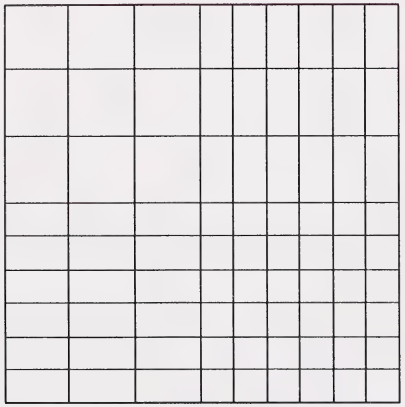


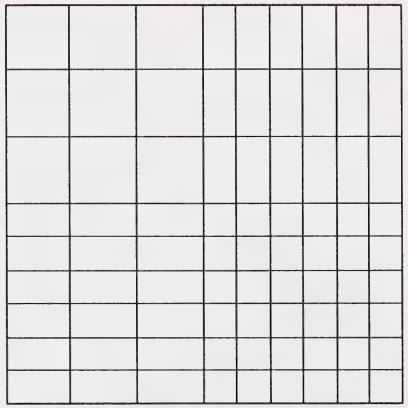


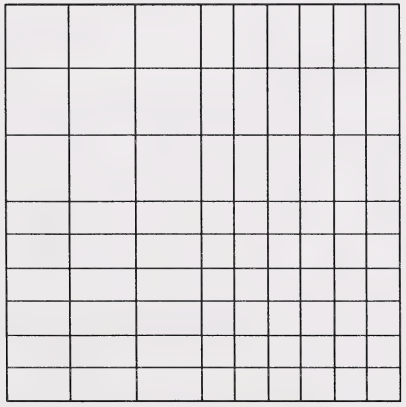


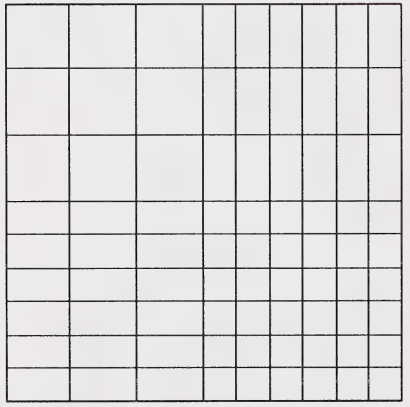


Binomial Grids

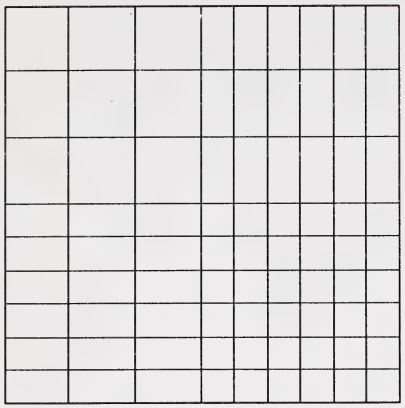


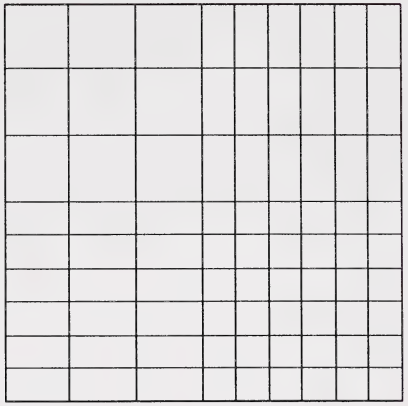


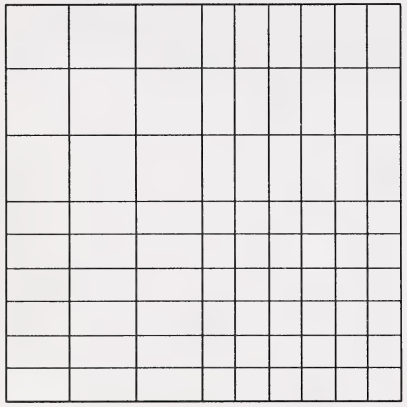


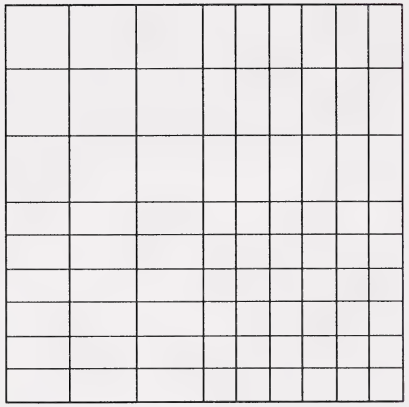


Binomial Grids

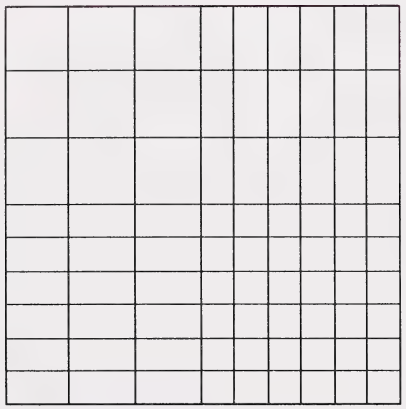


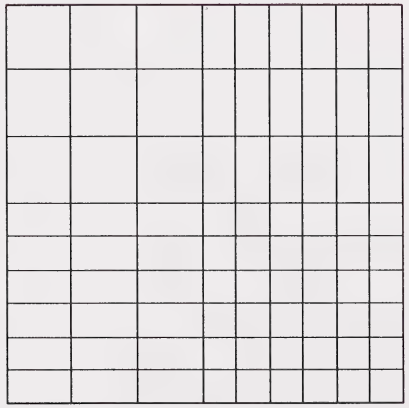


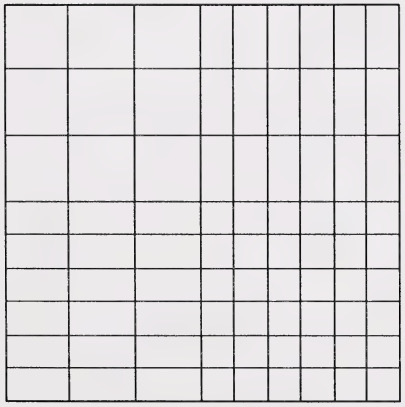


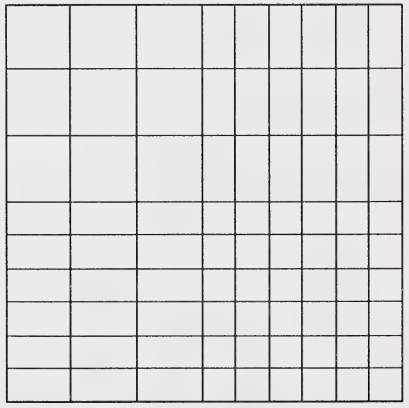


Binomial Grids

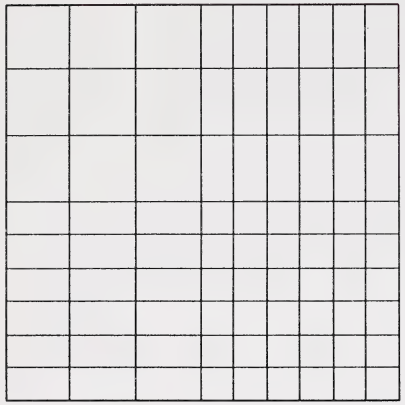


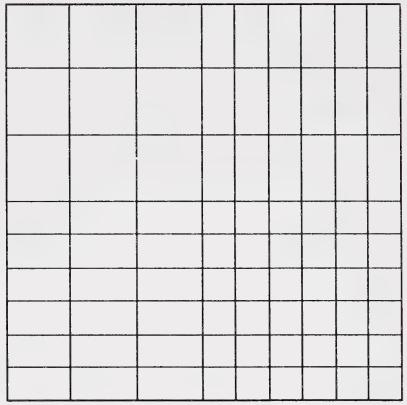


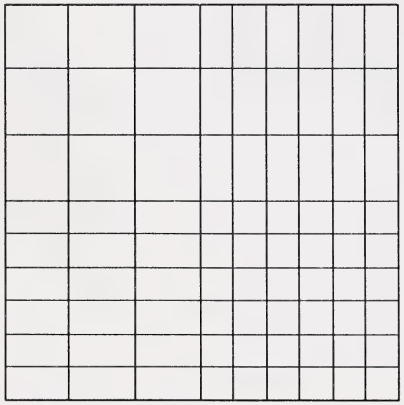


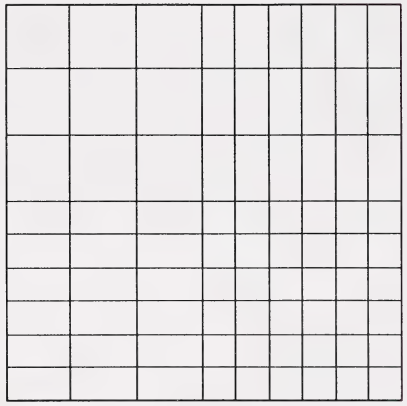


Binomial Grids

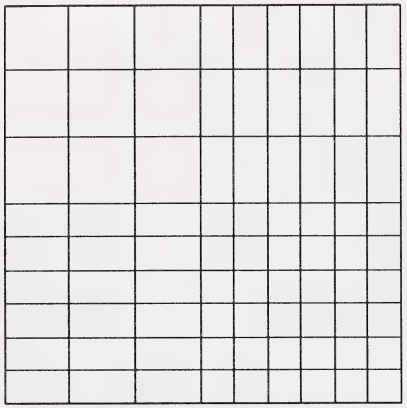


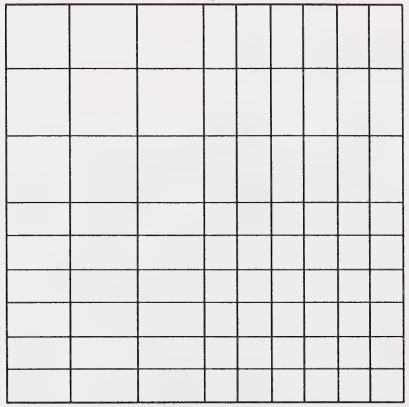


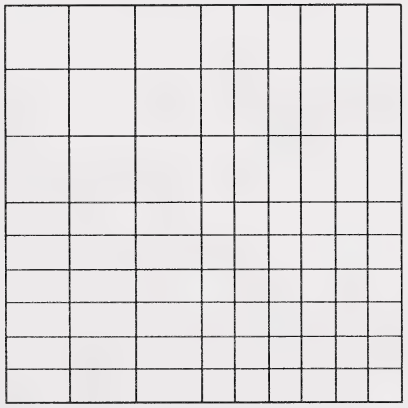


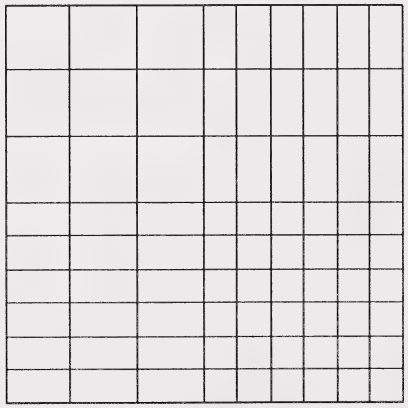


Binomial Grids

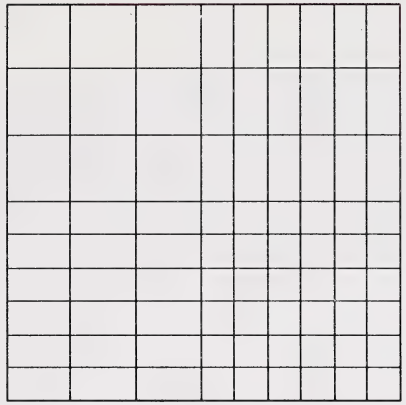


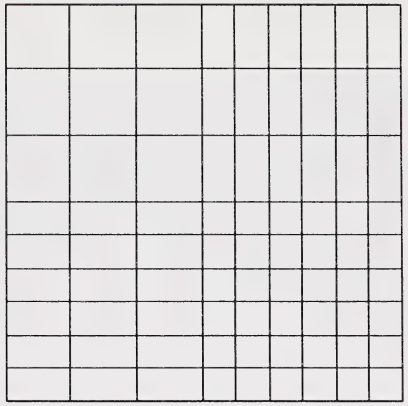


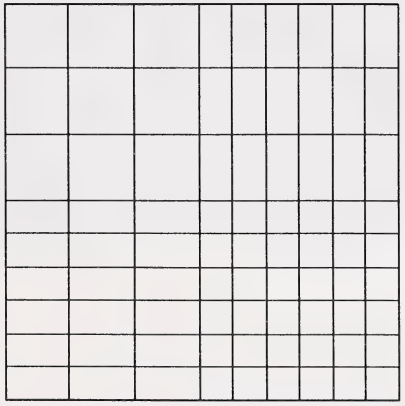


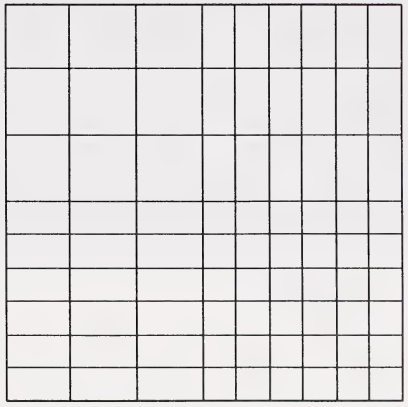


Binomial Grids

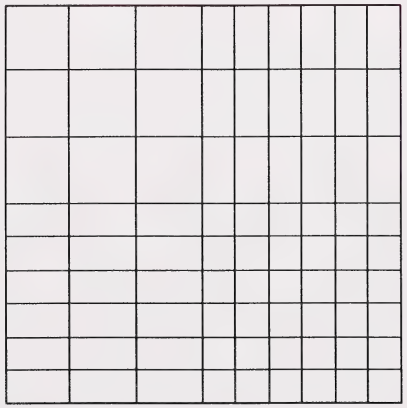


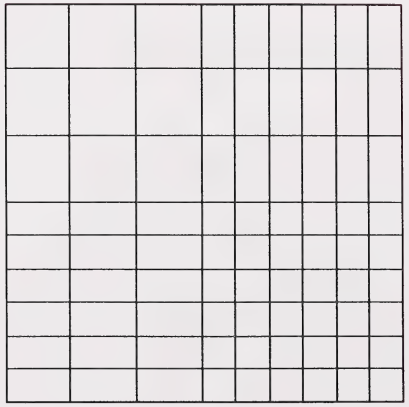


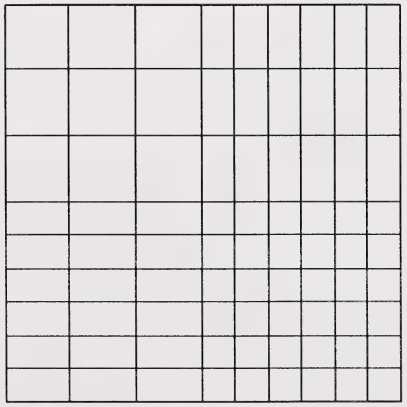


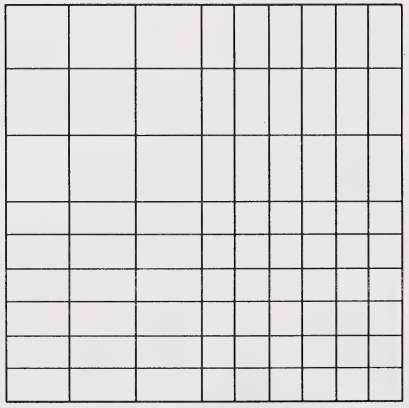


Binomial Grids

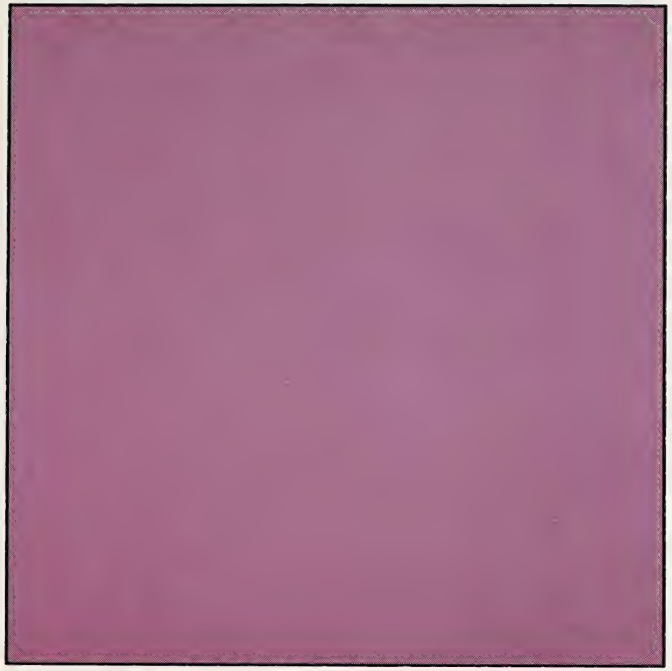


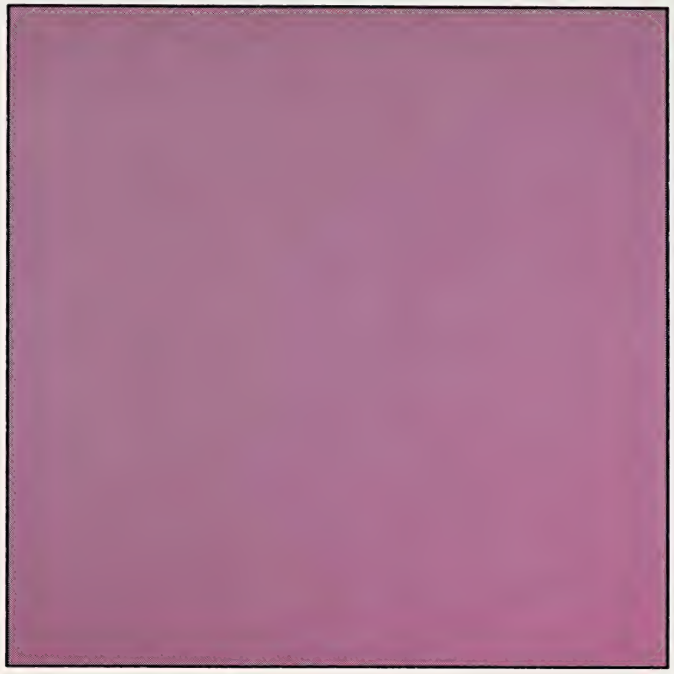






Polynomial Tiles

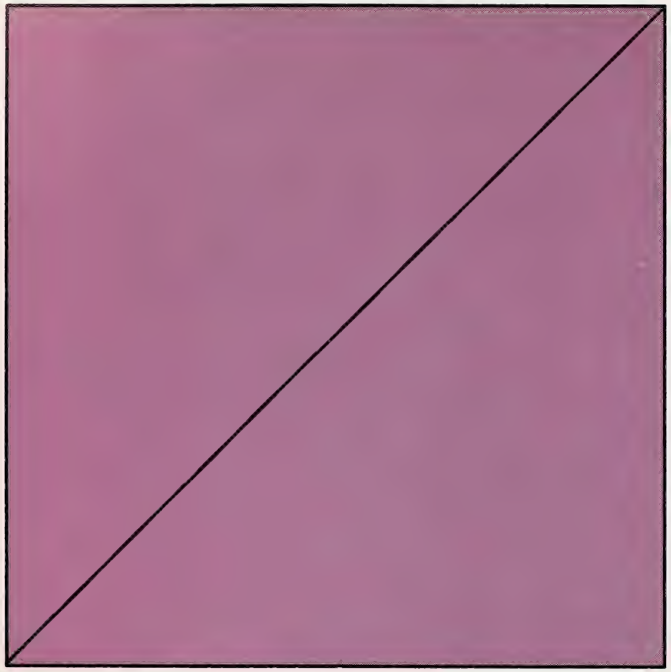


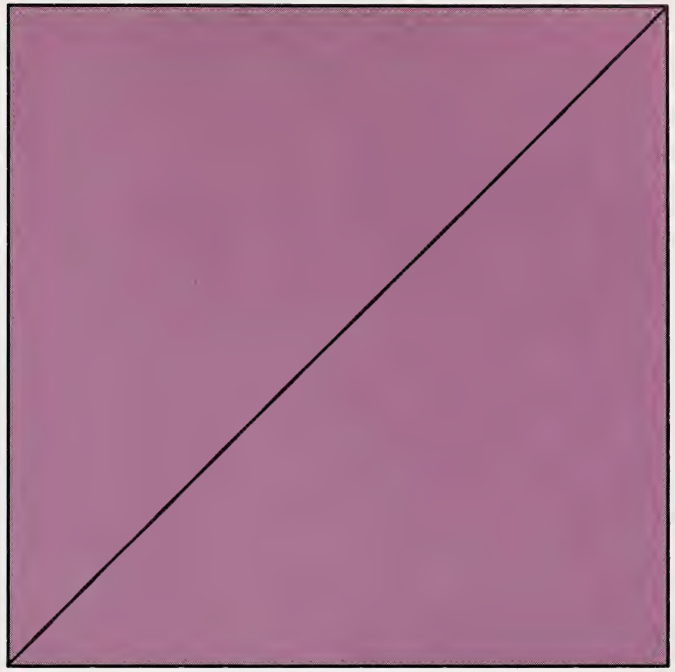


Each of these squares represents $+x^2$. Each one is called a flat.

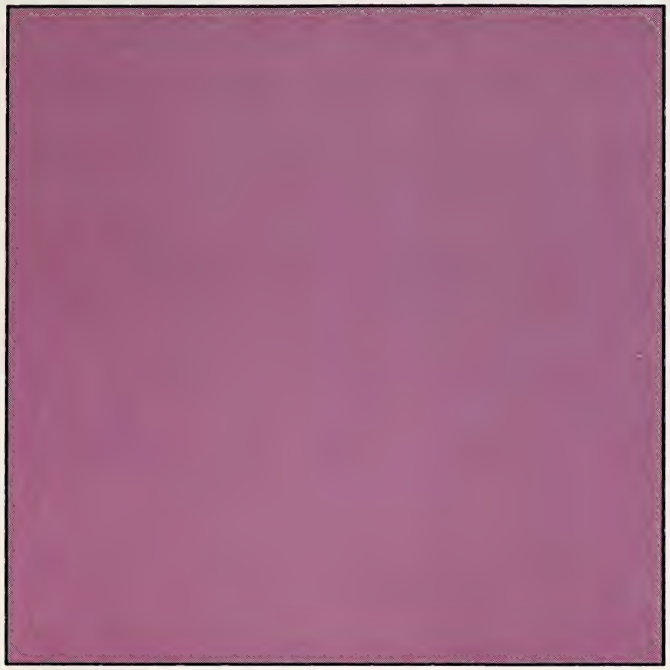
Polynomial Tiles

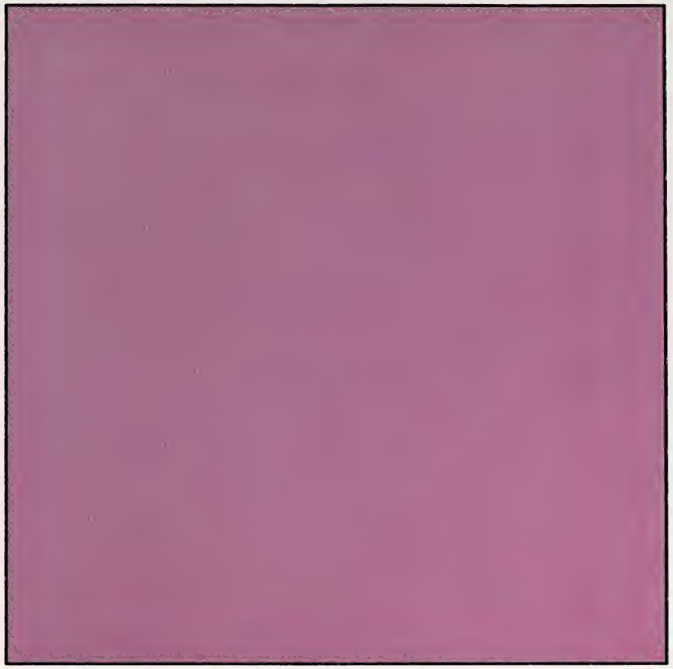
Each of these squares represents $-x^2$. Each one is called a negative flat.





Polynomial Tiles

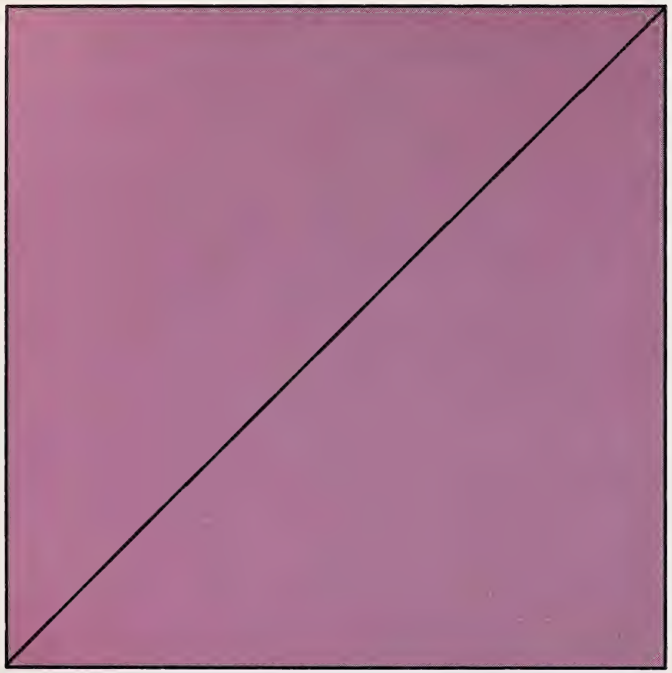


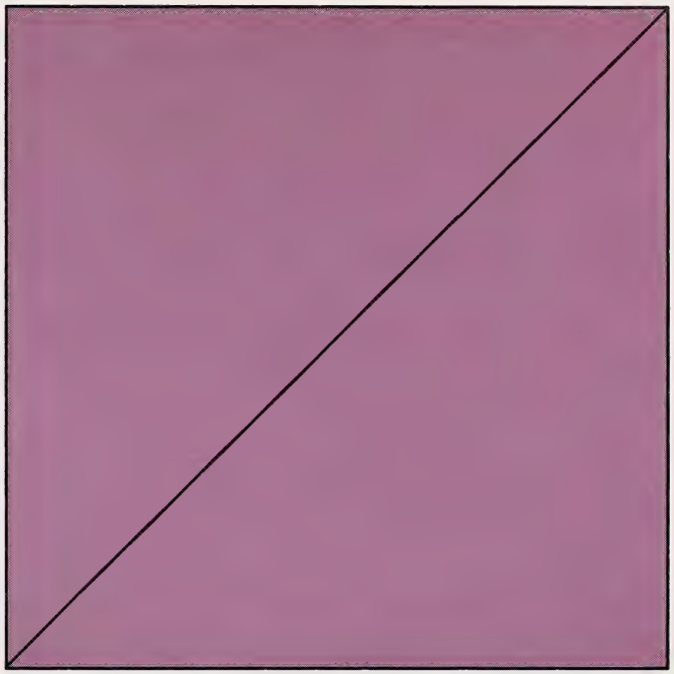


Each of these squares represents $+x^2$. Each one is called a flat.

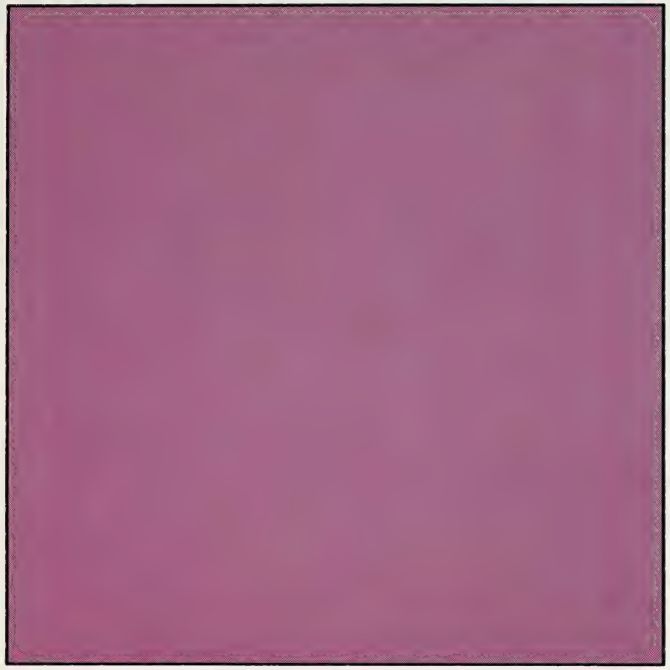
Polynomial Tiles

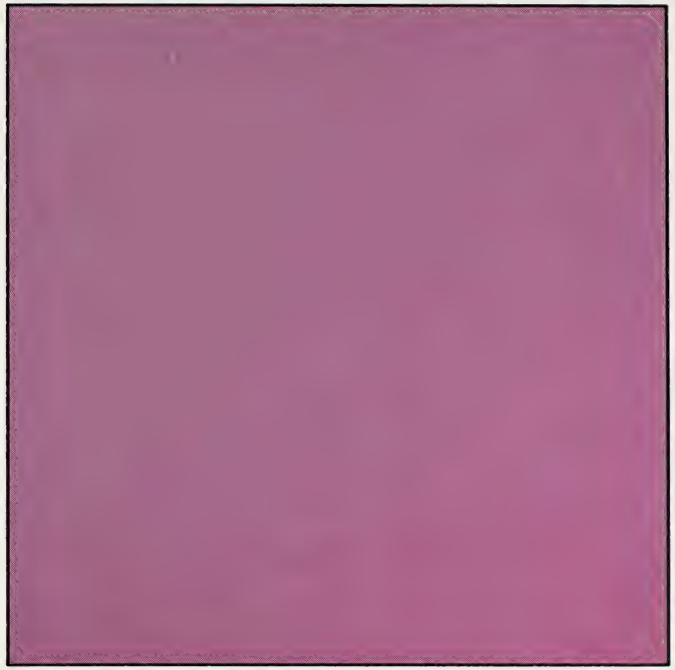
Each of these squares represents $-x^2$. Each one is called a negative flat.





Polynomial Tiles

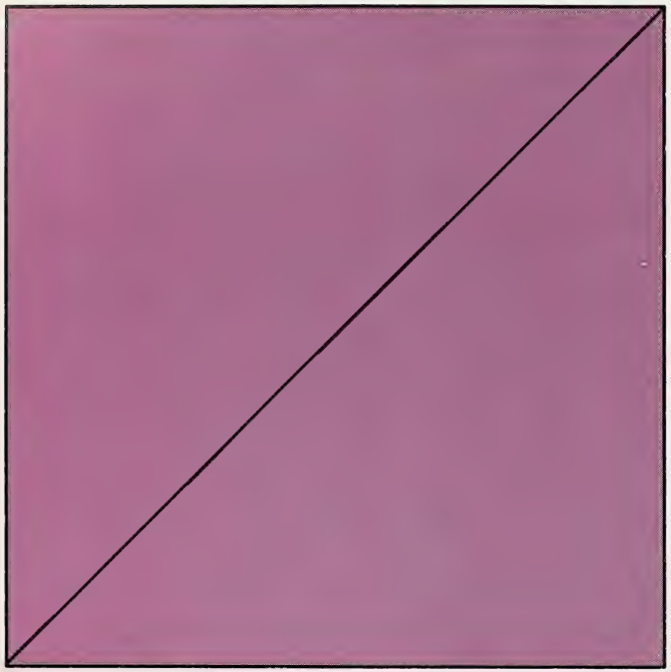


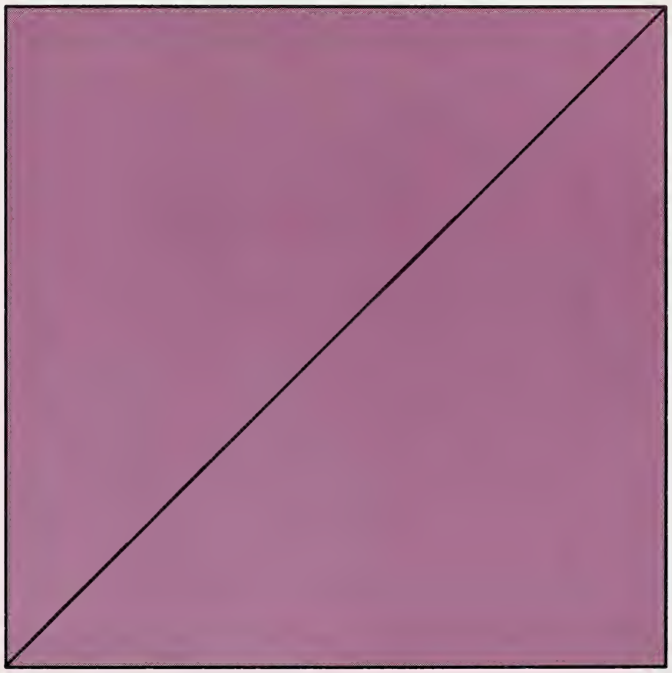


Each of these squares represents $+x^2$. Each one is called a flat.

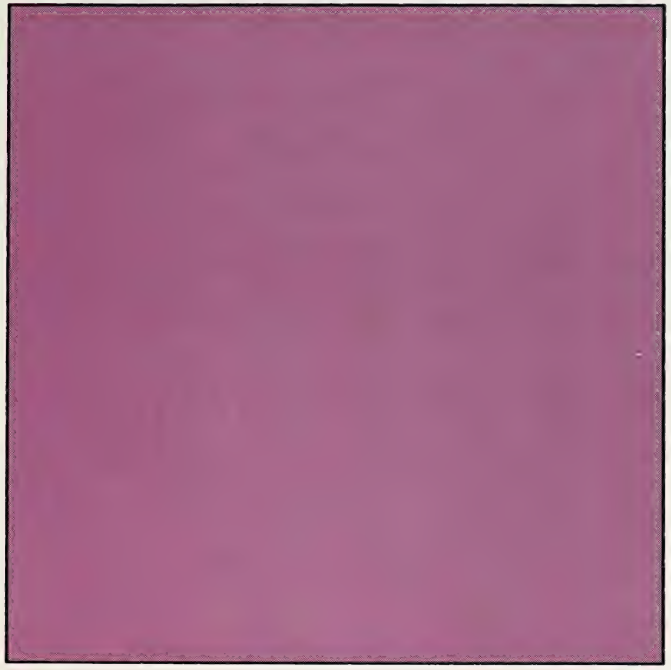
Polynomial Tiles

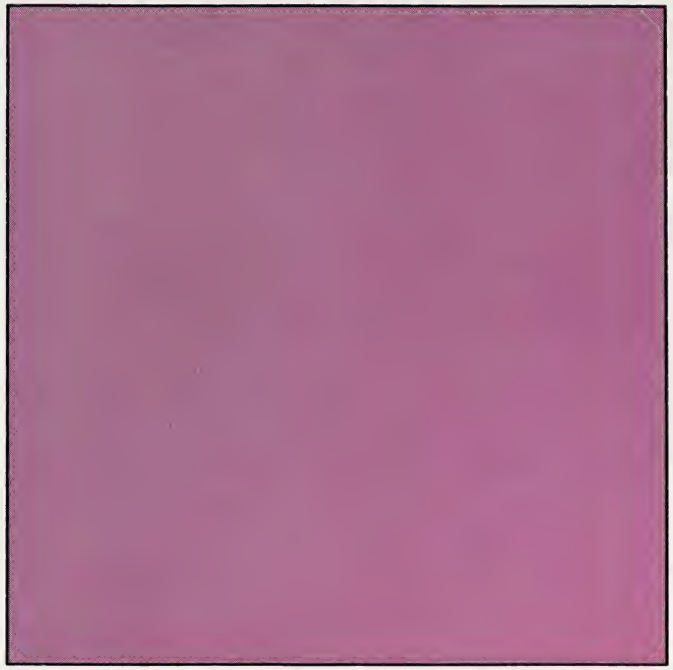
Each of these squares represents $-x^2$. Each one is called a negative flat.





Polynomial Tiles

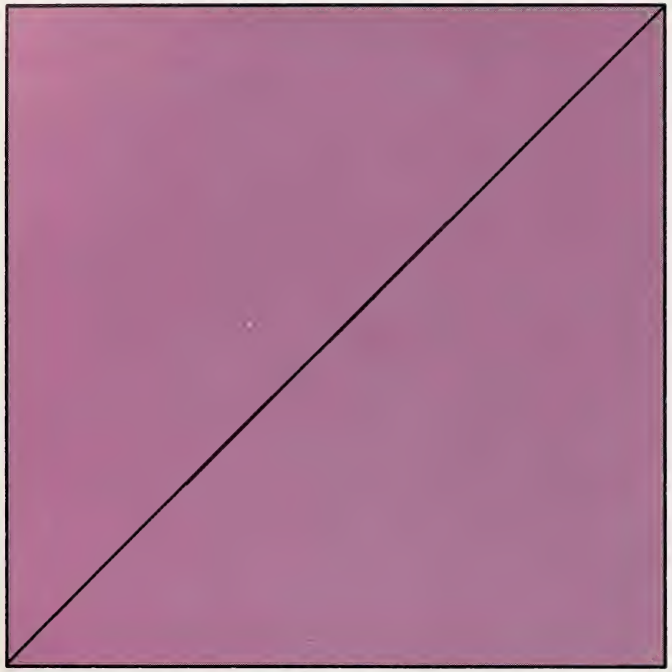


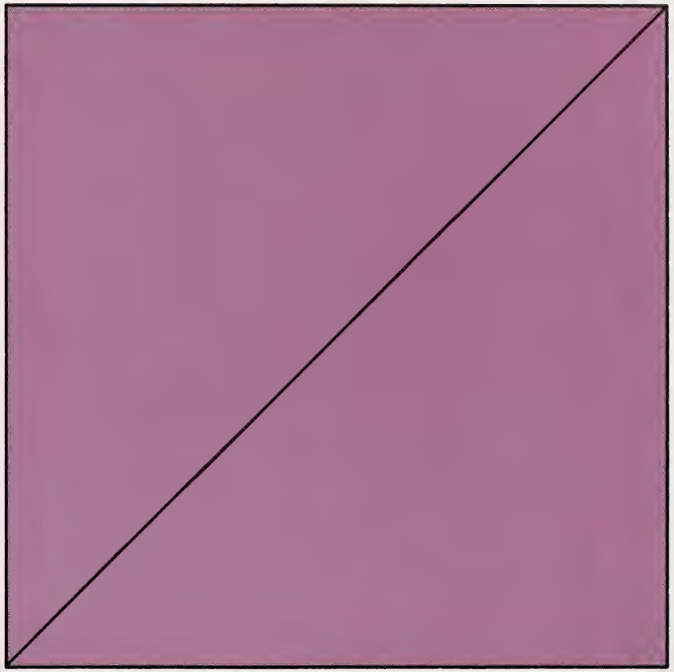


Each of these squares represents $+x^2$. Each one is called a flat.

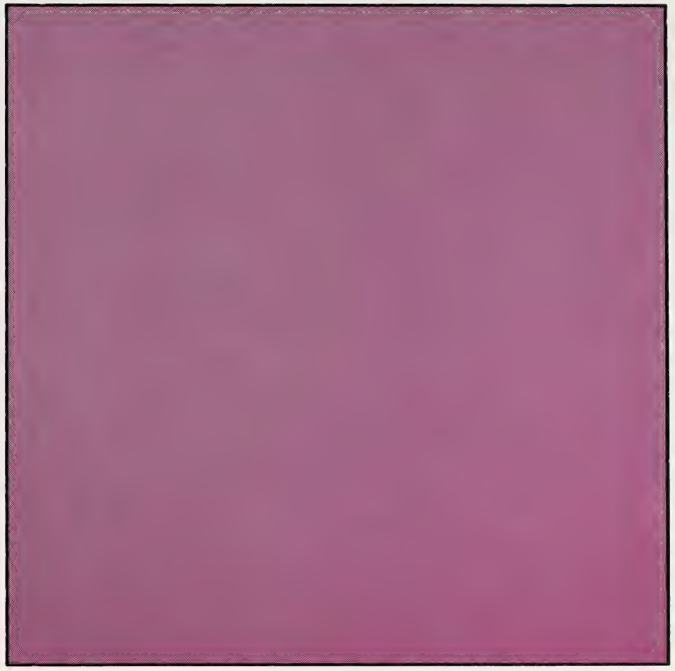
Polynomial Tiles

Each of these squares represents $-x^2$. Each one is called a negative flat.





Polynomial Tiles



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Polynomial Tiles

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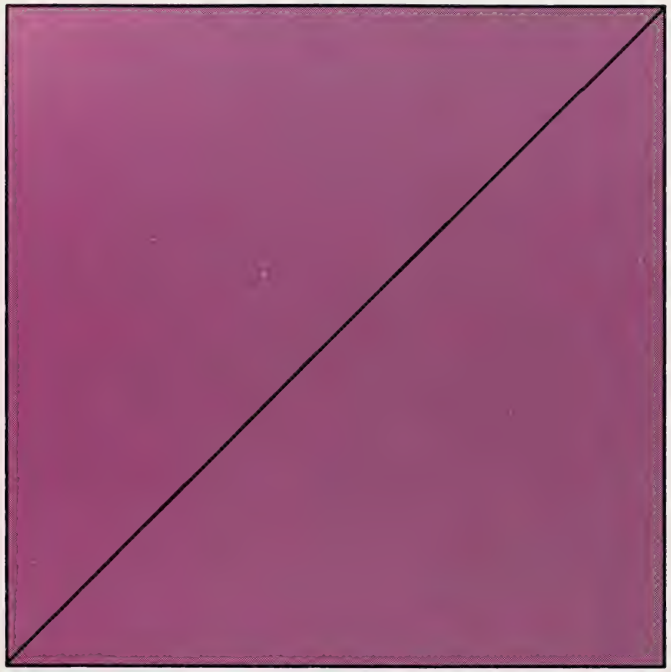
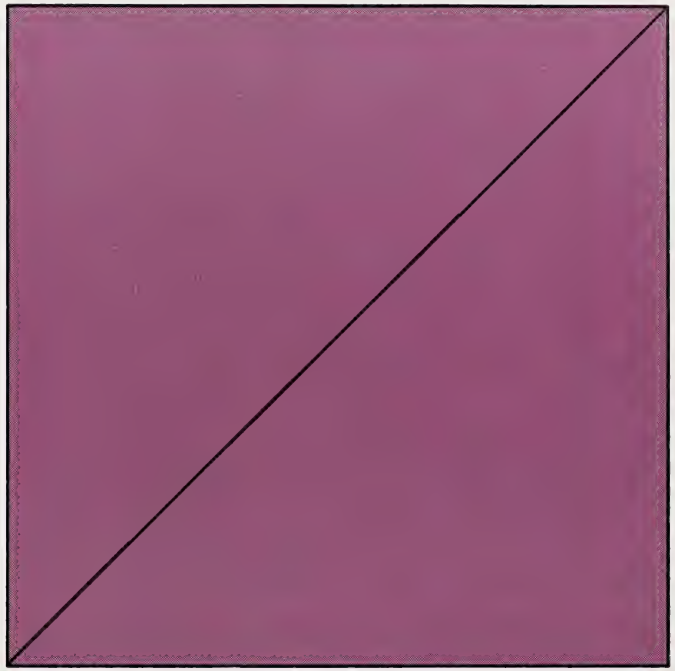
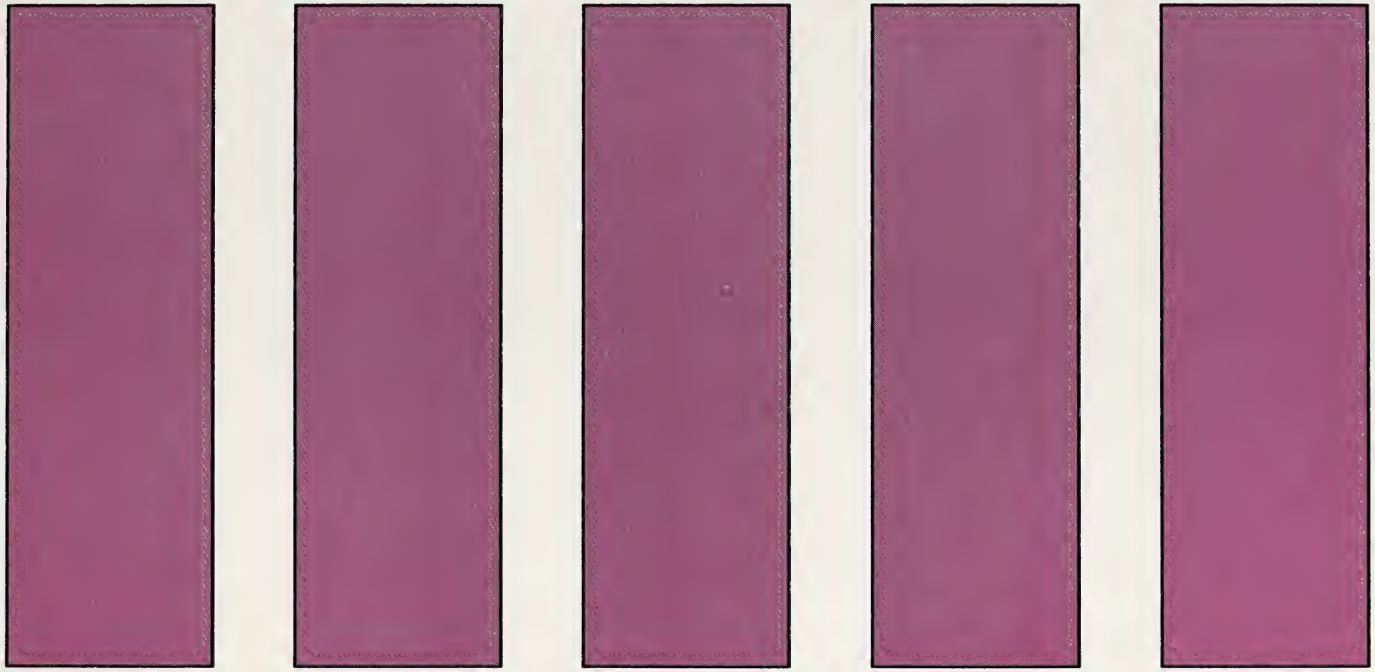




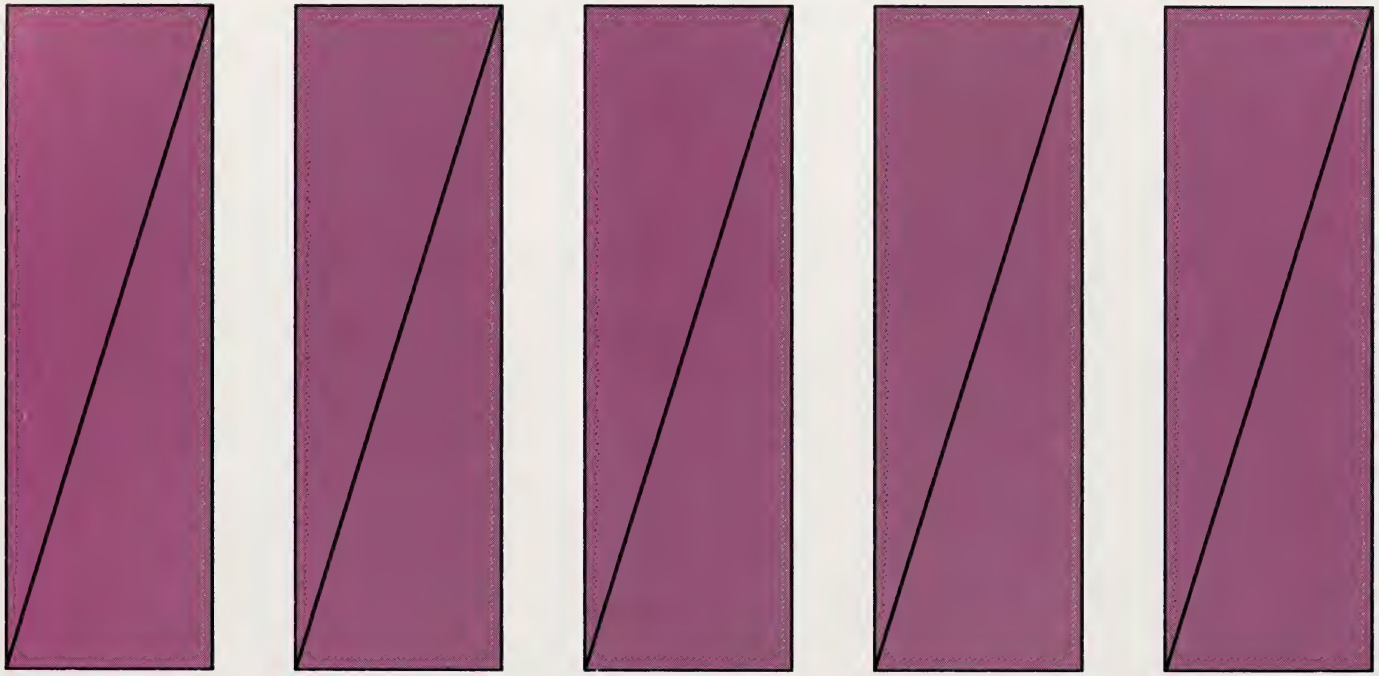
Polynomial Tiles



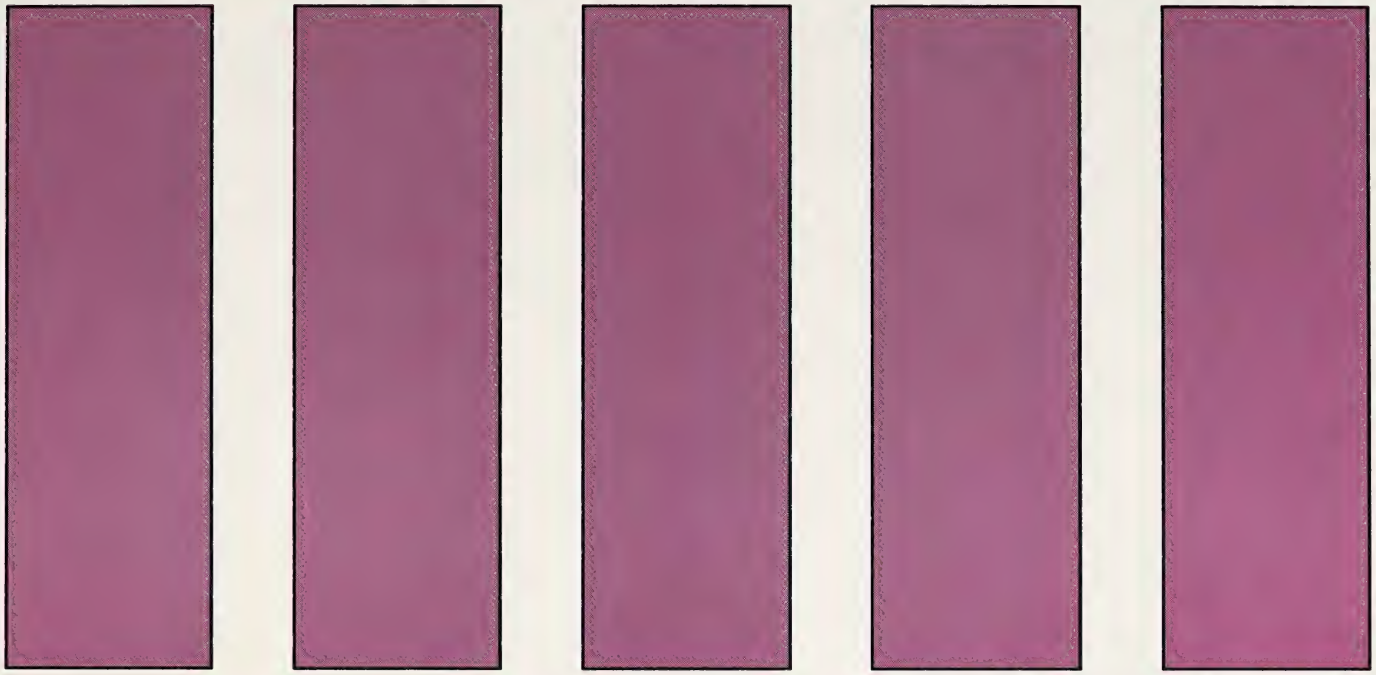
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



Polynomial Tiles



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Polynomial Tiles

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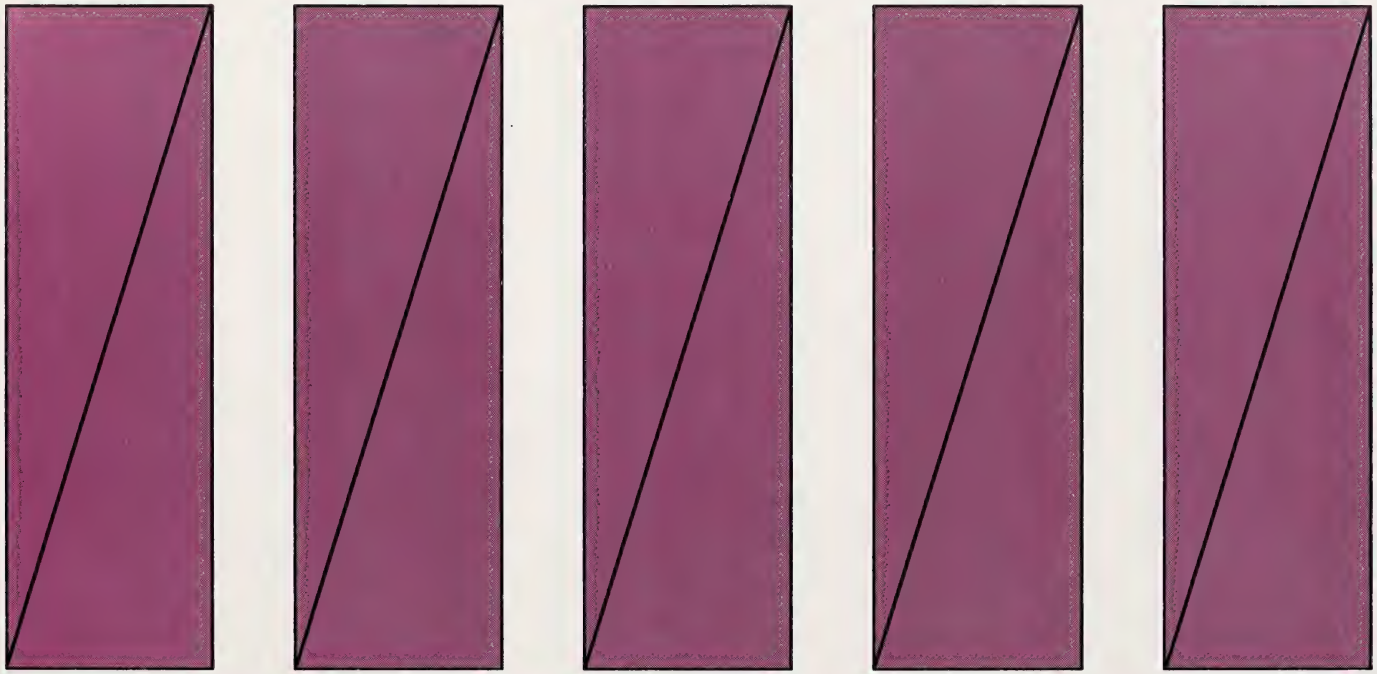
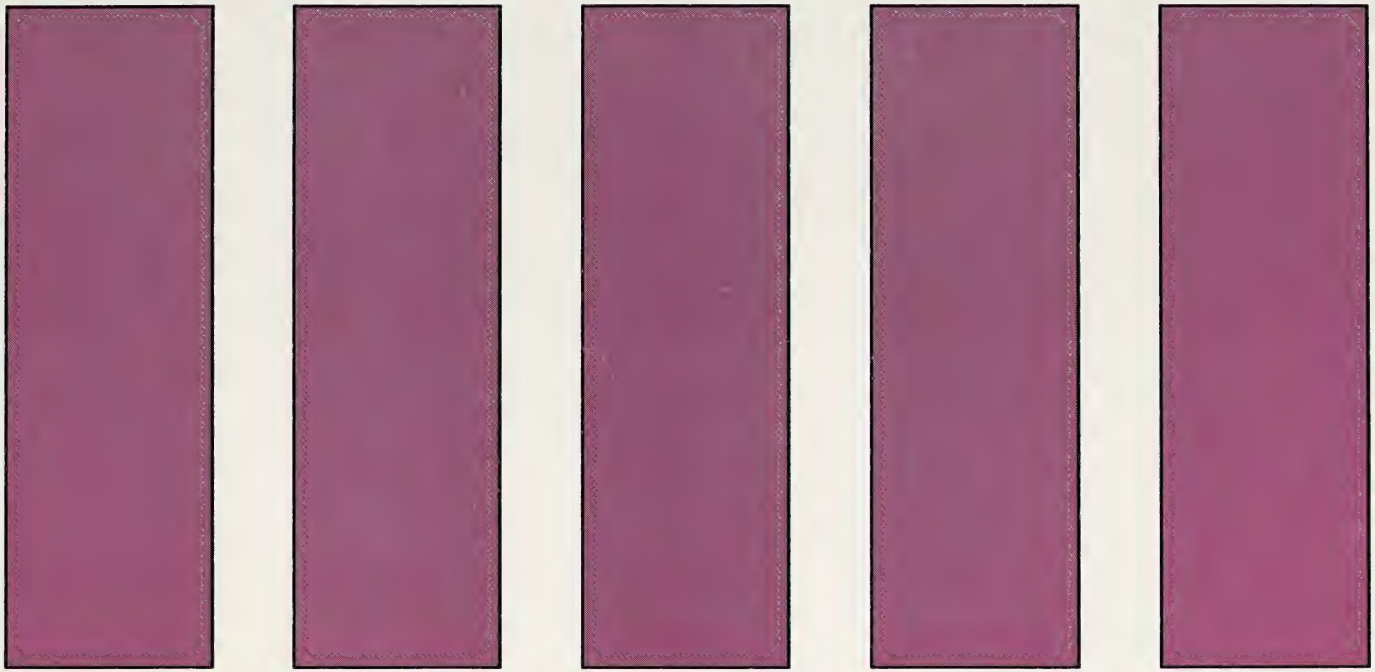


Polynomial Tiles



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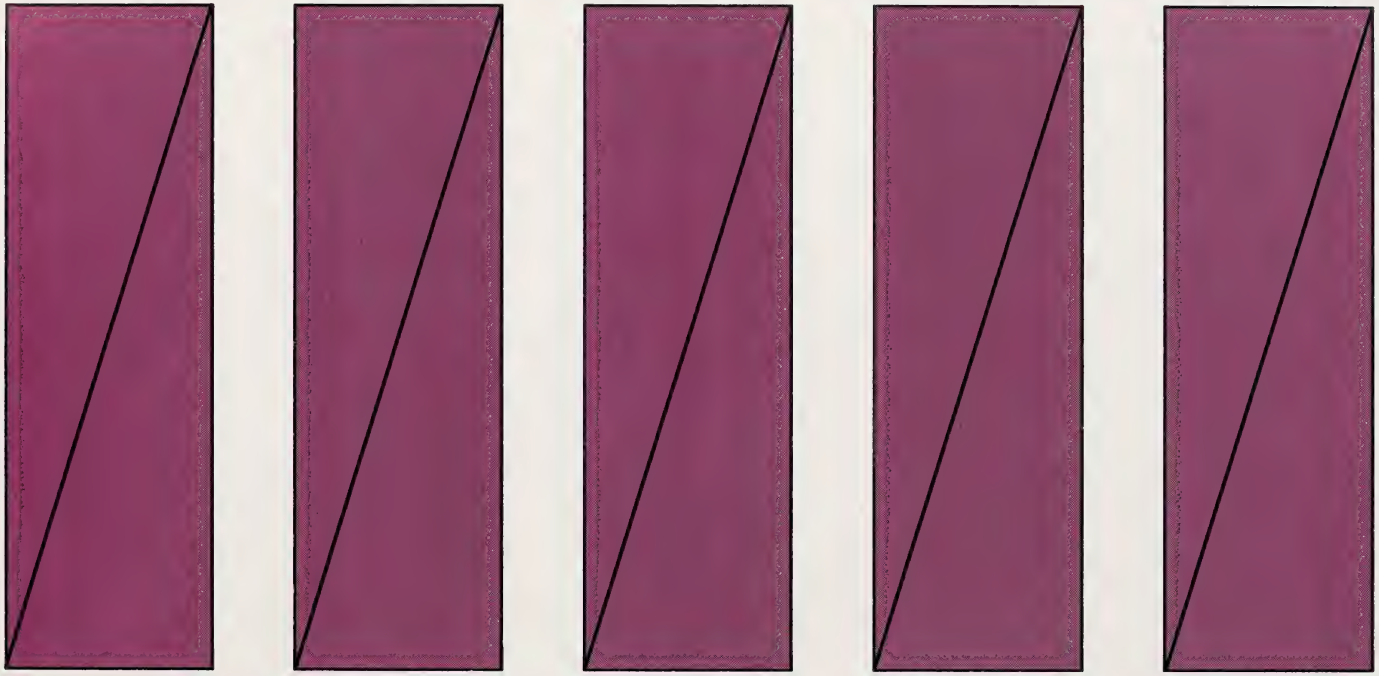
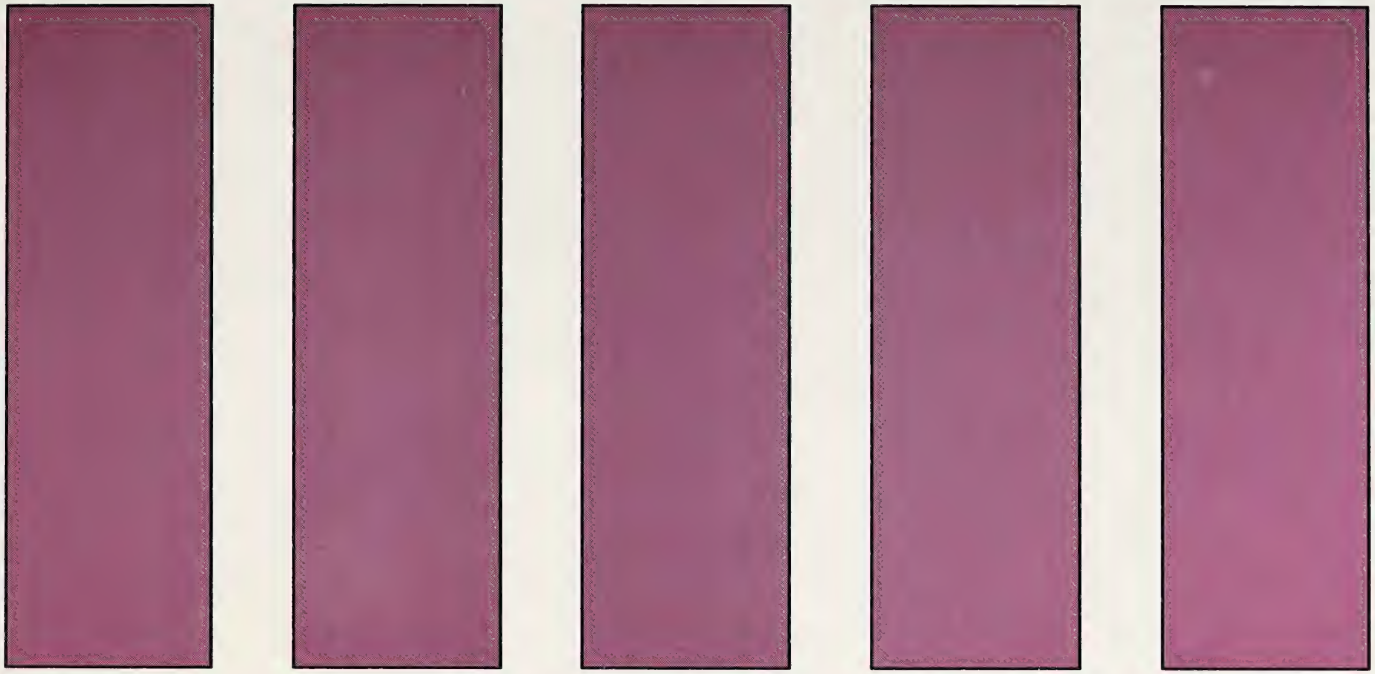


Polynomial Tiles



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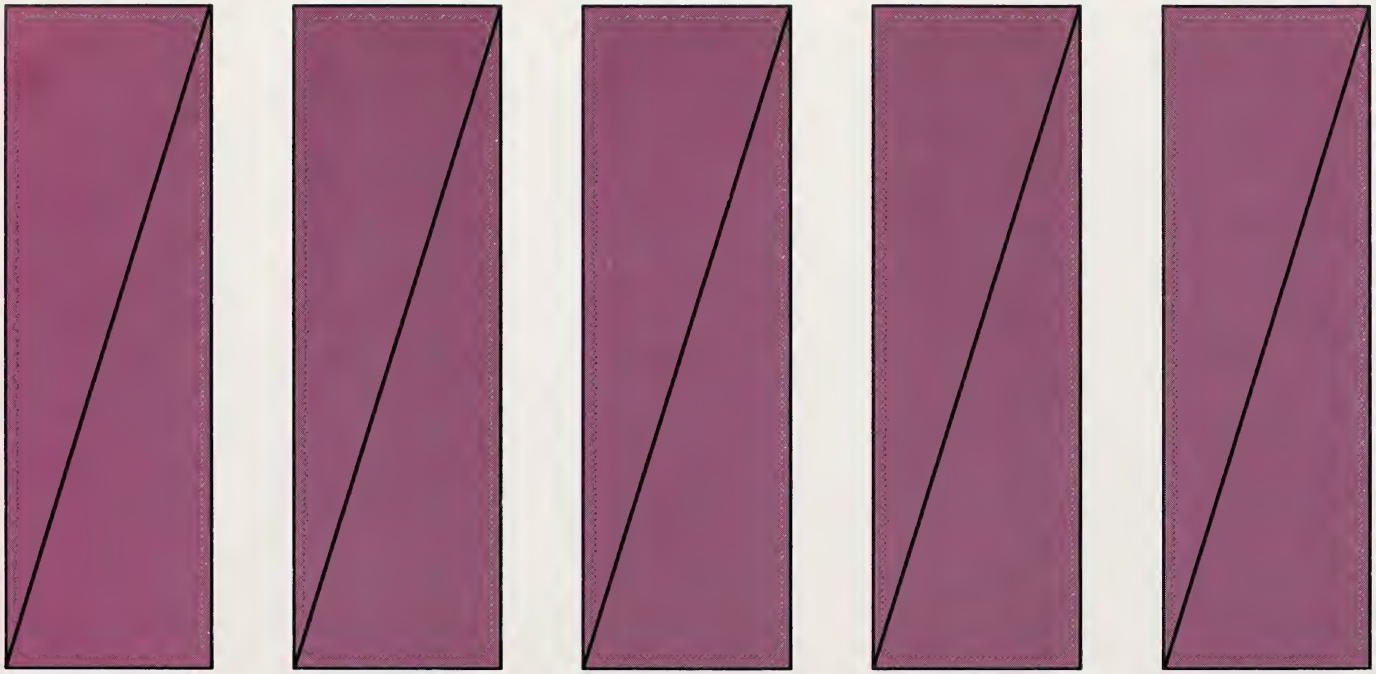
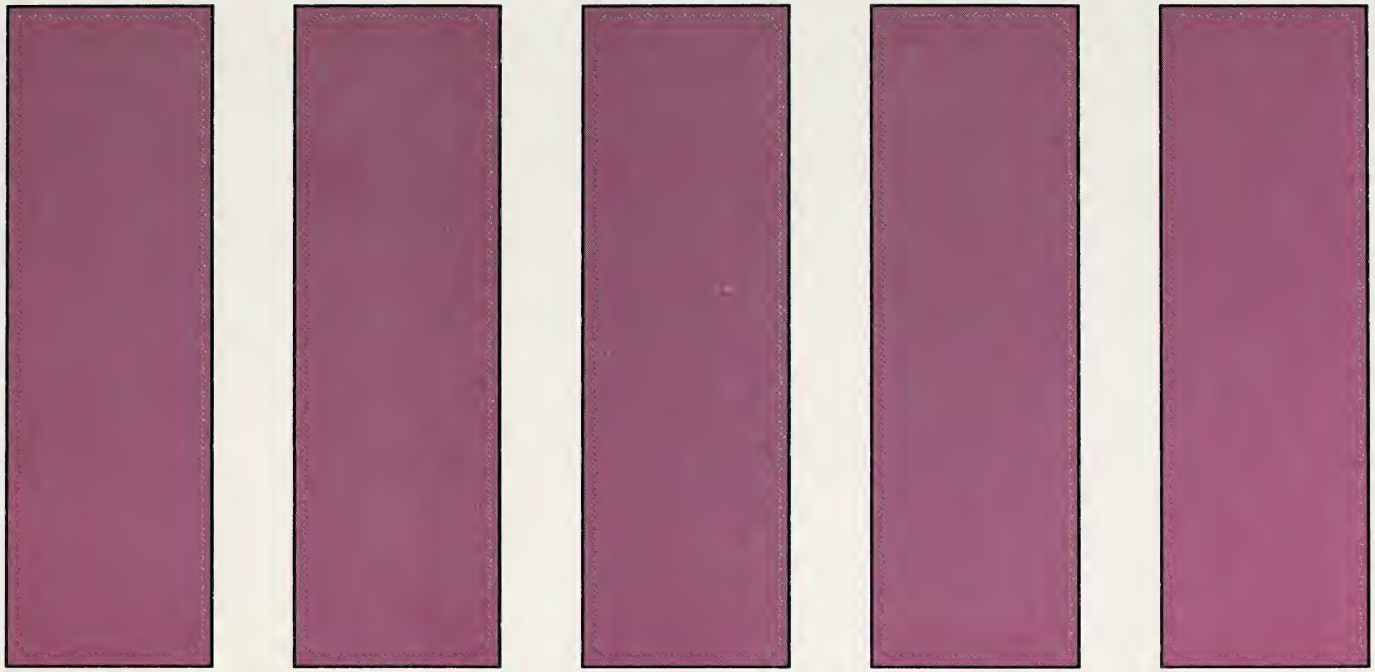
Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



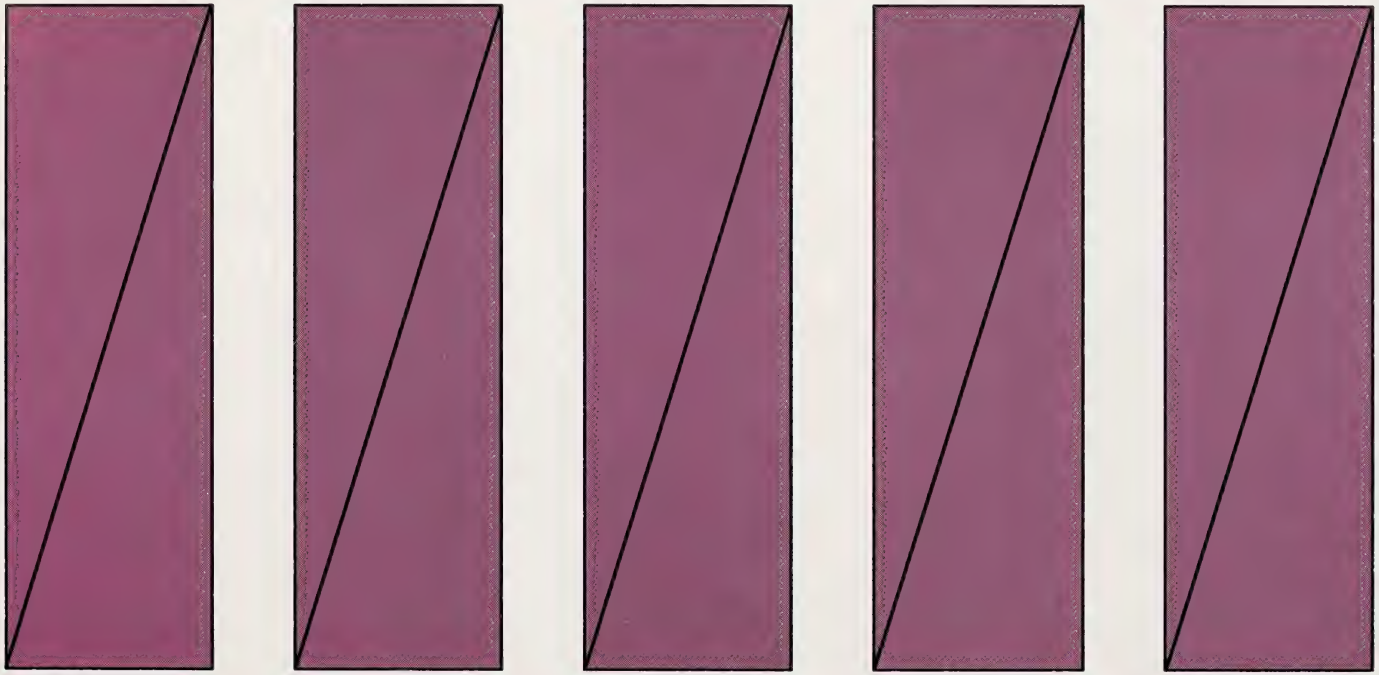
Polynomial Tiles



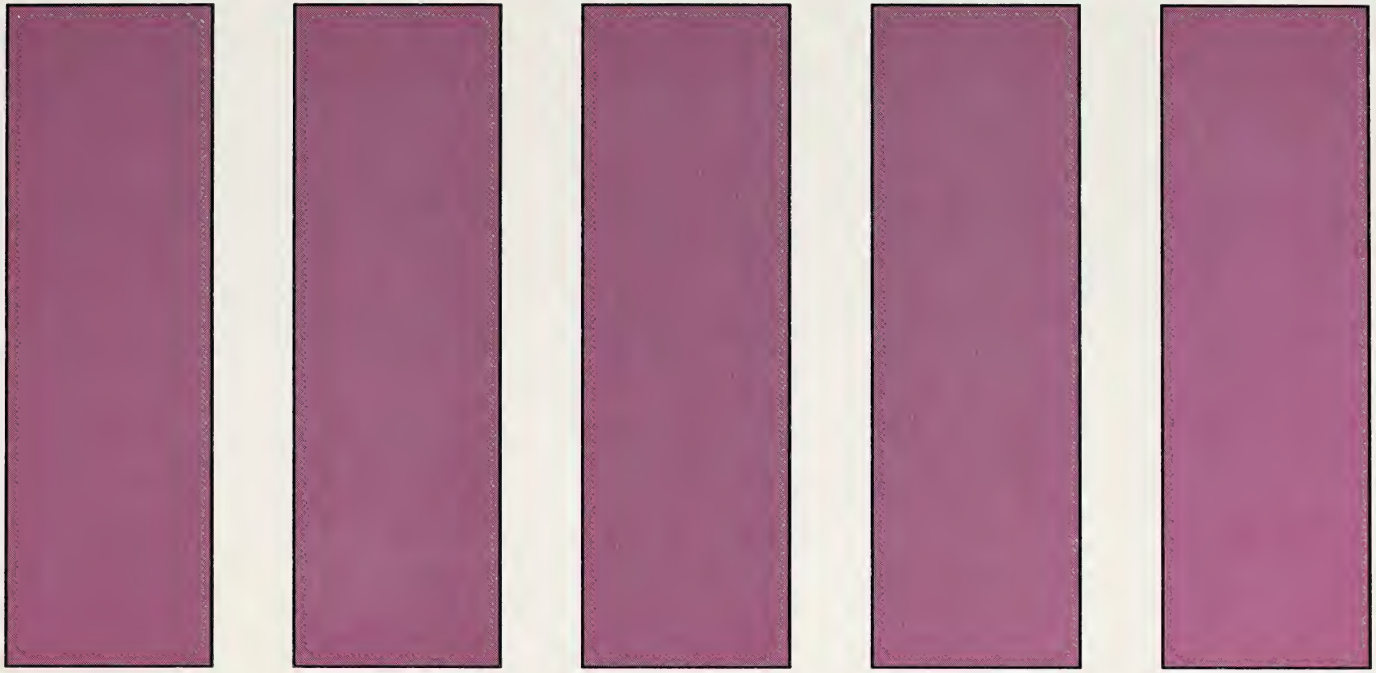
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



Polynomial Tiles



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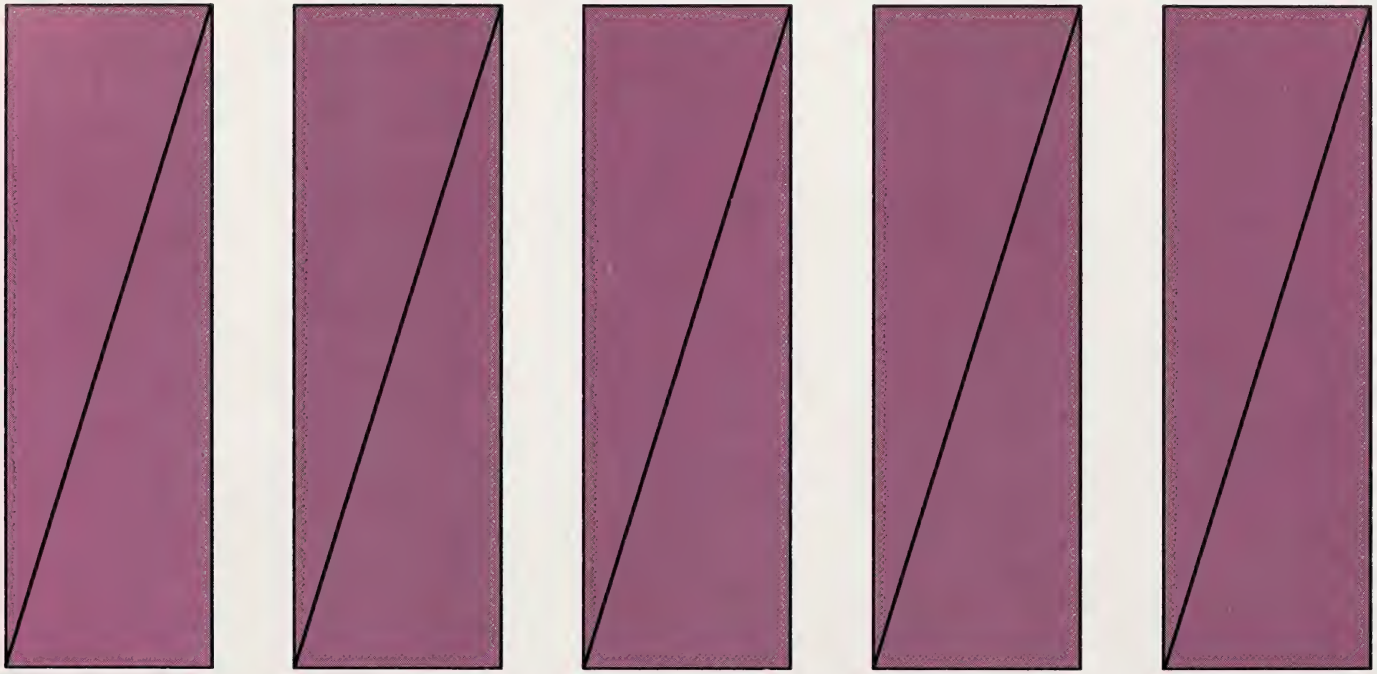
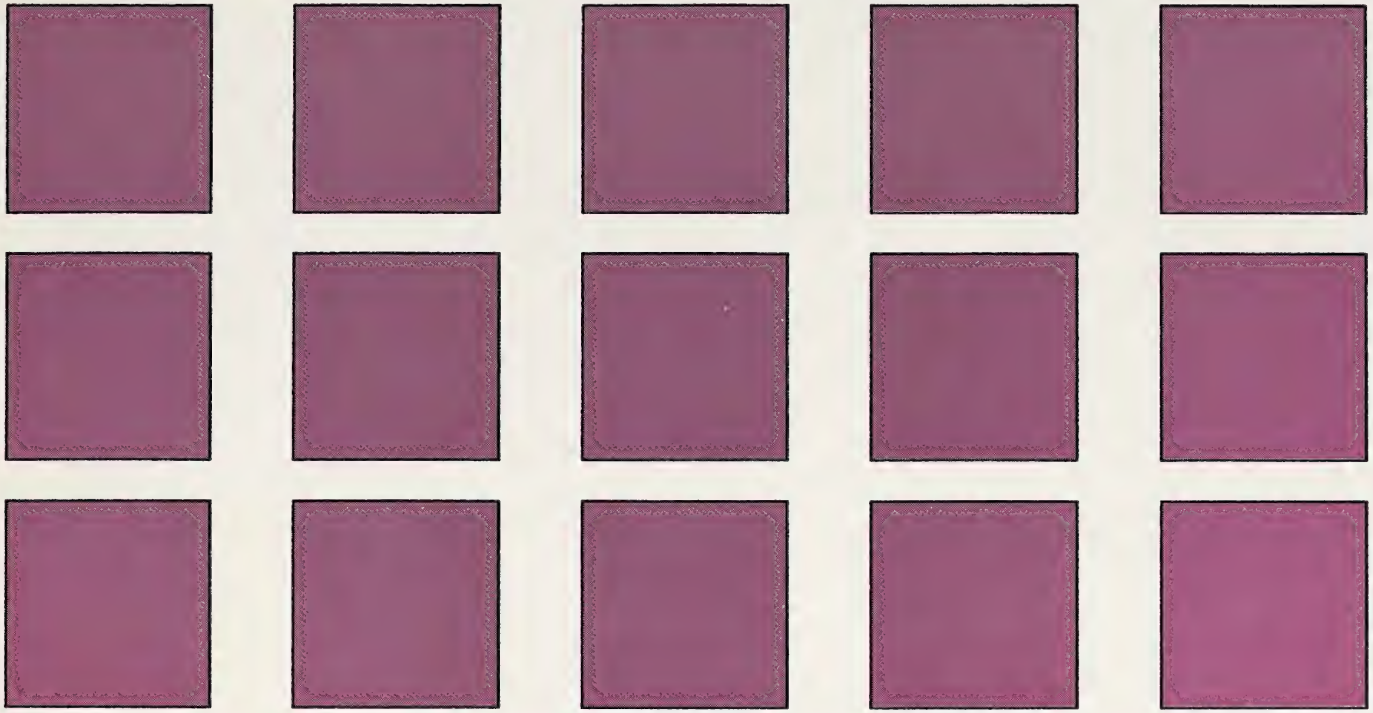


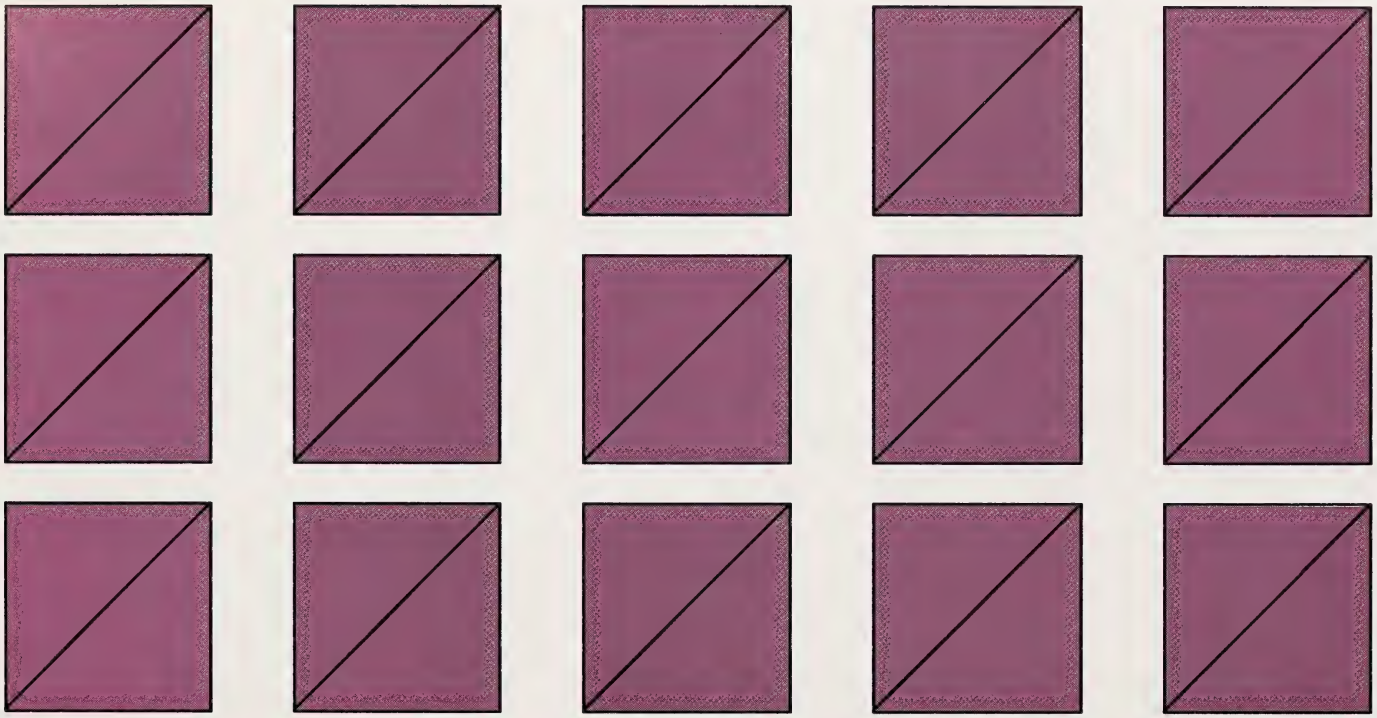
Polynomial Tiles



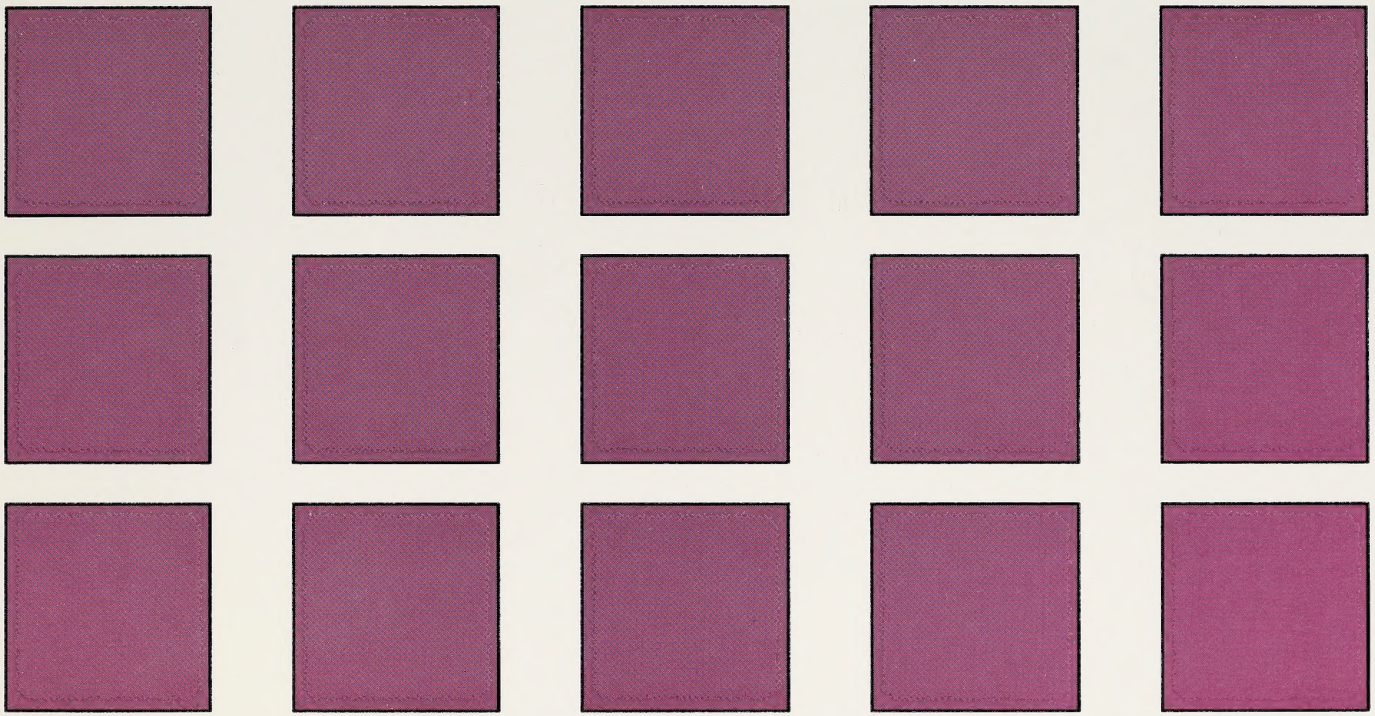
Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.



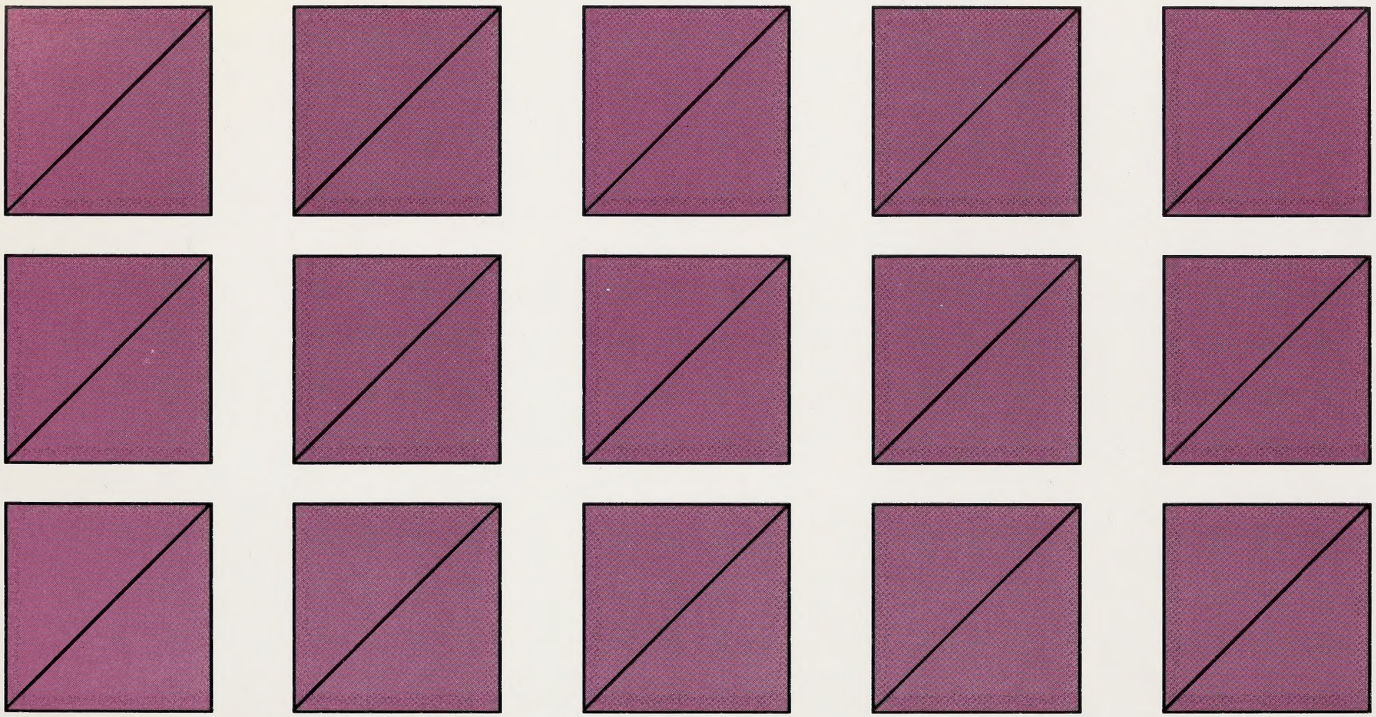
Polynomial Tiles



Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.



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L.R.D.C.
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Mathematics 10

Student Module
Unit 4

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